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Instability of an Unbounded Elastic Plate in a Gas Flow

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Abstract — The stability of an unbounded plane elastic plate in gas moving on one side of the plate and at rest on the other is analyzed. The gases are inviscid and in general different. The plate is under tension and has flexural stiffness. It is shown that the system is always unstable to plane sinusoidal perturbations with wave vector parallel to the velocity. As limiting cases, a tangential discontinuity between the two gases and unilateral flow past a plate with constant pressure on the opposite side are considered. In these cases, the conditions of stability to plane perturbations are non-trivial and are investigated below.

Keywords: plate stability, flow past a plate, plate flutter, tangential discontinuity, dispersion equation, stability criterion, linear stability theory.

The problem of the stability of a plane infinite plate in a unilateral gas flow with constant pressure on the other side was studied in [1, 2]. The main assumption of those studies was that the Mach number M is much greater than unity and the excess gas pressure on the plate can be found from the piston theory ([3], p. 261). In [1, 2], it was shown that this flow is always unstable. In this study we consider the problem without any assumptions about the order of M and construct the solution using the gas dynamics equations. It is shown that for each plate there exists a critical Mach number such that the flow is unstable for higher Mach numbers (which is in agreement with the results of [1, 2]) and stable for lower Mach numbers.

The case of gas at rest on the other side of the plate is investigated in our study for the first time. It is shown that this system is always unstable.

The limiting case of the absence of a plate (tangential discontinuity) was analyzed in [4] (p. 453–454) on the assumption that the parameters of the top and bottom gases are the same. In the present study, this case is qualitatively investigated for different gases on each side of the plate.

The stability of a plane membrane (flexural stiffness $D = 0$) with supersonic gas flow on one side and constant pressure on the other, in the form of a strip infinite in the direction perpendicular to the flow, was investigated in [5] (outlined in [6], pp. 245–250). It was shown that as $\mathrm{M} \to \infty$ the system is stable for arbitrarily large (but finite) plate width. This is consistent with the result obtained below, since the stability conditions for unbounded systems are in general different from those for arbitrarily extensive bounded systems [7].

Certain mechanisms neglected in our study may also lead to instability. For example, instability may develop due to shear stresses applied to the plate [8]. Taking into account the fluid viscosity [9–12] or dissipation in the plate [13–15] may lead to an expansion of the instability region.

1. FORMULATION OF THE PROBLEM

We will investigate in the linear approximation the stability of a plane thin elastic plate, infinite in all directions, below which lies a gas at rest, while another gas flows above and parallel to the plate at a constant velocity u. We will assume the gases to be ideal and perfect with densities ρ_1 and ρ_2 (subscripts "1" and "2" relate to the upper and lower gases, respectively) and speeds of sound a_1 and a_2 and the flow to be adiabatic. The plate is subjected to isotropic tension and has flexural stiffness. The plate surface density ρ_w, the tensile force N and the cylindrical stiffness D are assumed to be constant. There are no mass forces.

We will fix the coordinate system xyz so that the x and y axes lie in the plane of the unperturbed plate, the x axis being directed along the velocity u vector, whereas the z axis is perpendicular to the plate and directed upward.

Let a small perturbation independent of the y coordinate be superimposed on the system. We introduce the following notation: $\varphi_j(x, z, t)$ is the velocity perturbation potential of gas j and $w(x, t)$ is the plate deflection. The linearized equations and boundary conditions can then be written in the form:

$$\begin{aligned}
&\frac{\partial^2\varphi_1}{\partial t^2} + (u^2 - a_1^2)\frac{\partial^2\varphi_1}{\partial x^2} + 2u\frac{\partial^2\varphi_1}{\partial x\partial t} - a_1^2\frac{\partial^2\varphi_1}{\partial z^2} = 0, \qquad z > 0\\
&\frac{\partial^2\varphi_2}{\partial t^2} - a_2^2\frac{\partial^2\varphi_2}{\partial x^2} - a_2^2\frac{\partial^2\varphi_2}{\partial z^2} = 0, \qquad z < 0\\
&\frac{\partial\varphi_1}{\partial z} = u\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}, \qquad \frac{\partial\varphi_2}{\partial z} = \frac{\partial w}{\partial t}, \qquad z = 0\\
&\rho_w\frac{\partial^2 w}{\partial t^2} = \rho_1\left(\frac{\partial\varphi_1}{\partial t} + u\frac{\partial\varphi_1}{\partial x}\right) - \rho_2\frac{\partial\varphi_2}{\partial t} + N\frac{\partial^2 w}{\partial x^2} - D\frac{\partial^4 w}{\partial x^4}, \qquad z = 0\\
&\frac{\partial\varphi_1}{\partial q} = 0, \quad z \to +\infty, \qquad \frac{\partial\varphi_2}{\partial q} = 0, \quad z \to -\infty \qquad (q = x, z, t)
\end{aligned} \tag{1.1}$$

In what follows we will consider perturbations of the form $\varphi_j = f_j(z)e^{i(kz - \omega t)}$, $w = Ce^{i(kz - \omega t)}$, where k is the real wave number and ω is the complex frequency. Then, from (1.1) we obtain the following dispersion equation:

$$(\rho_w\omega^2 - Nk^2 - Dk^4)\sqrt{k^2 - \left(\frac{\omega}{a_2}\right)^2}\sqrt{k^2 - \left(\frac{\omega - uk}{a_1}\right)^2} + \rho_2\omega^2\sqrt{k^2 - \left(\frac{\omega - uk}{a_1}\right)^2} + \rho_1(\omega - uk)^2\sqrt{k^2 - \left(\frac{\omega}{a_2}\right)^2} = 0 \tag{1.2}$$

Since the solutions decrease as $|z| \to \infty$, we can choose the square root branches in such a way that their real parts are positive. All the dynamic characteristics of the plate enter into the first term of Eq. (1.2).

We will nondimensionalize (1.2) taking a_1, ρ_1, and ρ_w as dimensionally independent quantities. Then

$$k = \frac{\rho_1}{\rho_w}k', \ \omega = \frac{q_1\rho_1}{\rho_w}\omega', \ u = \mathrm{M}a_1, \ N = a_1^2\rho_w\mathrm{M}_w^2, \ D = \frac{a_1^2\rho_w^3}{\rho_1^2}D', \ a_2 = \chi a_1, \ \rho_2 = \kappa\rho_1$$

The parameter M_w is the ratio of the long-wave propagation velocity in the plate $\sqrt{N/\rho_w}$ to the speed of sound a_1.

Omitting the primes on the dimensionless variables in Eq. (1.2) and making the substitution $\omega = kc$ (c is the phase perturbation velocity), we obtain the relation

$$k(c^2 - \mathrm{M}_w^2 - Dk^2)\sqrt{\chi^2 - c^2}\sqrt{1 - (c - \mathrm{M})^2} + \kappa\chi c^2\sqrt{1 - (c - \mathrm{M})^2} + (c - \mathrm{M})^2\sqrt{\chi^2 - c^2} = 0 \tag{1.3}$$

This equation depends on five dimensionless parameters: M, M_w, D, κ, and χ and has the following property: if a complex number c is a root of Eq. (1.3), then so is the conjugate number. The stability analysis will be based on the following assertion: if for at least one real k there exists a complex root c with nonzero imaginary part, the system is unstable; conversely, if all the roots are real, the system is stable.

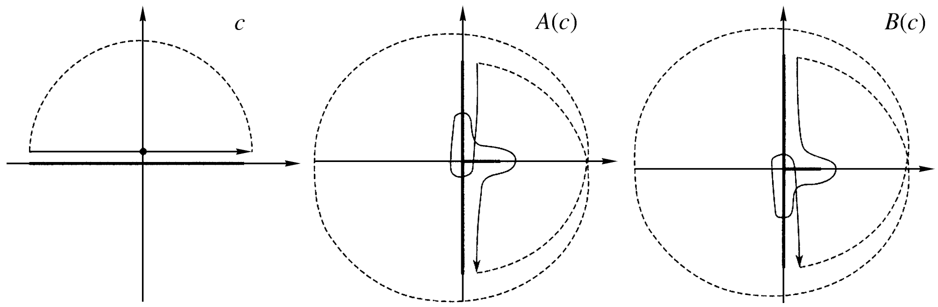

Fig. 1. Contour l in the planes c, $A(c)$, and $B(c)$

2. STABILITY OF A TANGENTIAL DISCONTINUITY

We will first assume that in the initial formulation there is no plate, $\kappa \neq 0$, and $\chi \neq 0$ and will thus investigate a tangential discontinuity between the gases. The dispersion equation corresponding to this case can be obtained from (1.3) by discarding the first term which alone reflects the plate effect:

$$\kappa\chi c^2\sqrt{1-(c-\mathrm{M})^2}+(c-\mathrm{M})^2\sqrt{\chi^2-c^2}=0 \tag{2.1}$$

Equation (2.1) depends only on three parameters M, χ, and κ and is independent of k. This means that for a tangential discontinuity all perturbations are stable or unstable simultaneously and there is no dispersion.

We rewrite Eq. (2.1) in the form:

$$A(c)+B(c)=0, \quad A(c)=\kappa\chi c^2\sqrt{1-(c-\mathrm{M})^2}, \quad B(c)=(c-\mathrm{M})^2\sqrt{\chi^2-c^2}$$

We will now consider in the complex half-plane $\operatorname{Im} c>0$ the closed curve l consisting of the upper bank of the real axis (that is, the real axis shifted by $i\varepsilon$ as $\varepsilon \to +0$) and an infinitely large semicircle with center at the point $c=0$. Since (2.1) has a finite number of roots in the upper half-plane, all these roots lie inside l. Due to the fact that on and inside this curve all the functions entering into the left side of (2.1) are analytical, we can find the number of roots using the argument principle. We will represent the semicircle by a broken curve, the real axis by a bold line, and the direction of the real axis circuit by an ordinary curve. The curve l and the possible form of its images under the action of $A(c)$ and $B(c)$ are shown in Fig. 1. The images of the real axis lie on the real and imaginary axes and the mappings have the following structure:

$$A(c):\ [-\infty;\ \mathrm{M}-1]\to[+i\infty;\ 0],\ [\mathrm{M}-1;\ \mathrm{M}+1]\to[0;\ \alpha],\ \alpha>0,\ [\mathrm{M}+1;\ +\infty]\to[0;\ -i\infty]$$

$$B(c):\ [-\infty;\ -\chi]\to[+i\infty;\ 0],\ [-\chi;\ \chi]\to[0;\ \beta],\ \beta>0, \quad [\chi;\ +\infty]\to[0;\ -i\infty]$$

Thus, the function $A(c)+B(c)$ maps the interval $(-\infty;\ \min\{-\chi,\ \mathrm{M}-1\}]$ into an interval on the positive imaginary semiaxis and the interval $[\max\{-\chi,\ \mathrm{M}+1\};\ \infty)$ into an interval on the negative imaginary semiaxis; hence, only the image of the segment $[\min\{-\chi,\ \mathrm{M}-1\};\ \max\{\chi,\ \mathrm{M}+1\}]$ can lie in the neighborhood of zero and determine the presence of complex roots.

We will consider two cases. Let the segments $[-\chi;\ \chi]$ and $[\mathrm{M}-1;\ \mathrm{M}+1]$ intersect. Then the segment $[\min\{-\chi,\ \mathrm{M}-1\};\ \max\{\chi,\ \mathrm{M}+1\}]$ consists of their union, which means that the image of an arbitrary point of this segment under the action of the function $A(c)+B(c)$ is the sum of two numbers: one positive real and the other positive real or purely imaginary. Therefore, the image of the entire segment lies in the right half-plane and the image of the curve l looks as shown in Fig. 2. This curve makes one turn around zero; hence, Eq. (2.1) has complex roots and in this case the system is unstable.

Now let the segments $[-\chi;\ \chi]$ and $[\mathrm{M}-1;\ \mathrm{M}+1]$ not intersect. Then the left side of (2.1) maps the segment $[\chi;\ \mathrm{M}-1]$ into a segment on the imaginary axis, χ and $\mathrm{M}-1$ being mapped into points with positive and negative imaginary parts, respectively. The image of the entire segment $[\chi;\ \mathrm{M}-1]$ passes

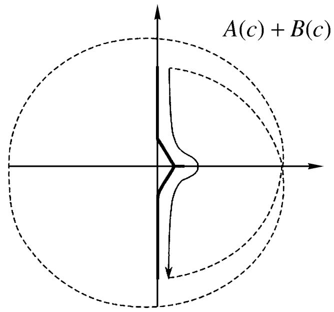

Fig. 2. Contour l in the plane of the function $A(c) + B(c)$ for $\mathrm{M} - 1 < \chi$

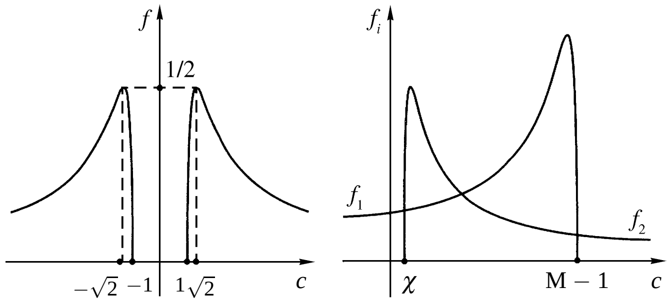

Fig. 3. Graphs of the functions $f(c)$, $f_1(c)$, and $f_2(c)$

through zero an odd number of times. This number cannot be greater than three, since otherwise the l image would turn around the coordinate origin a negative number of times and $A(c) + B(c)$ would have poles in the upper half-plane, which is not the case. If the image of $[\chi;\ \mathrm{M} - 1]$ passes through zero once, then (2.1) has complex roots and if three times, then all the roots are real. This is easy to see by counting up the number of l image turns around zero.

Taking into account the fact that on the segment $[\chi;\ \mathrm{M} - 1]$ the imaginary part of the first radical in (2.1) is positive and the imaginary part of the second is negative, we multiply (2.1) by i and obtain:

$$(c - \mathrm{M})^2\sqrt{c^2 - \chi^2} - \kappa\chi c^2\sqrt{(c - \mathrm{M})^2 - 1} = 0 \tag{2.2}$$

On the segment $[\chi;\ \mathrm{M} - 1]$, all the functions in (2.2) are real and all the square roots are positive. We convert (2.2) to the form:

$$f_1(c) = f_2(c), \quad f_1(c) = \kappa\chi^2 f(c - \mathrm{M}), \quad f_2(c) = f\left(\frac{c}{\chi}\right), \quad f(c) = \frac{\sqrt{c^2 - 1}}{c^2} \tag{2.3}$$

Since in the problem of the stability of a tangential discontinuity the top and bottom gases are equivalent (for them to change places it is necessary to go over to the coordinate system in which the upper gas is at rest), we can, without loss of generality, assume that $\chi = a_2/a_1 \leq 1$. Then the graphs of the functions $f(c)$, $f_1(c)$, and $f_2(c)$ look as shown in Fig. 3.

Above, using the argument principle, it was shown that if on the segment $[\chi,\ \mathrm{M} - 1]$ the graph of the function $y = f_1(c)$ intersects the graph of the function $y = f_2(c)$ at a single point, then (2.1) has complex roots, and if at three points, then all the roots of (2.1) are real. The following properties of Eq. (2.3) hold:

1. If $\mathrm{M} < \sqrt{2}\chi + 1$, the graph of $y = f_1(c)$ intersects the graph of $y = f_2(c)$ at a single point because on the segment $[\chi;\ \mathrm{M} - 1]$ the first function monotonically decreases and the second monotonically increases. This is a sufficient condition of monotony.

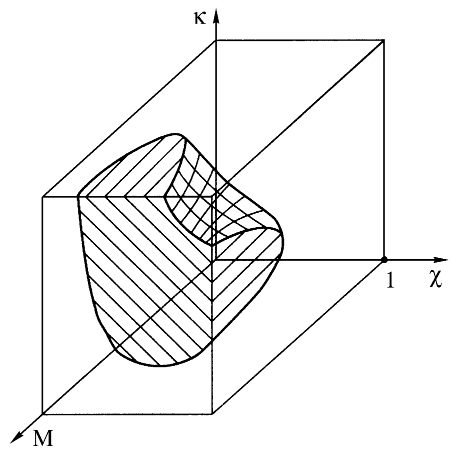

Fig. 4. Stability region of the tangential discontinuity in the space of the parameters (M; χ; κ) for $\chi \le 1$

2. The graph of $y = f_1(c)$ has a single intersection with the graph of $y = f_2(c)$ if $f_2(\mathrm{M} - 1) > \max f_1(c)$ or $f_1(\chi) > \max f_2(c)$. Solving these inequalities for M, we obtain another sufficient condition of instability:

$$\mathrm{M} < 1 + \sqrt{2}\frac{\sqrt{\sqrt{1-\kappa^2\chi^4}+1}}{\kappa\chi}, \qquad \kappa\chi^2 < 1$$

$$\mathrm{M} < \chi + \sqrt{2\kappa}\chi\sqrt{\sqrt{\kappa^2\chi^4 - 1} + \kappa\chi^2}, \qquad \kappa\chi^2 \ge 1$$

3. The graph of $y = f_1(c)$ intersects the graph of $y = f_2(c)$ at three points if $f_2(c_1) \le \max f_1(c) = f_1(c_1)$ and $f_1(c_2) \le \max f_2(c) = f_2(c_2)$. From this we obtain the sufficient stability condition

$$\mathrm{M} \ge \sqrt{2} + \sqrt{2}\frac{\sqrt{\sqrt{1-\kappa^2\chi^4}+1}}{\kappa\chi}, \qquad \kappa\chi^2 < 1$$

$$\mathrm{M} \ge \sqrt{2}\chi + \sqrt{2\kappa}\chi\sqrt{\sqrt{\kappa^2\chi^4 - 1} + \kappa\chi^2}, \qquad \kappa\chi^2 \ge 1$$

4. If for certain values of the parameters M_1, χ, and κ the graphs of $y = f_1(c)$ and $y = f_2(c)$ intersect at three points, then for parameters M_2, χ, and κ, such that $\mathrm{M}_2 > \mathrm{M}_1$ they also intersect at three points. This means that the stability region consists of half-lines parallel to the M axis and, in particular, is simply connected.

Using these four properties, we can qualitatively find the form of the stability region in the three-dimensional space of the parameters (M, χ, κ) as shown in Fig. 4.

Since at small M the tangential discontinuity is unstable, introducing perturbations aligned at an angle to the flow, we can always find growing perturbations among them. Thus, as in the case considered in [4], the tangential discontinuity is unstable to arbitrarily oriented perturbations.

3. STABILITY ANALYSIS IN THE PRESENCE OF A PLATE

We will now return to the study of the dispersion equation of the initial problem (1.3), assuming, as in the previous section, that $\kappa \ne 0$ and $\chi \ne 0$. We will investigate the asymptotic behavior of solutions of (1.3) as $k \to 0$, since even in this case complex roots exist for any problem parameters. The first term is small of the order of k; therefore, we will first consider Eq. (2.1) which is the dispersion equation for a tangential discontinuity and was previously analyzed in Section 2.

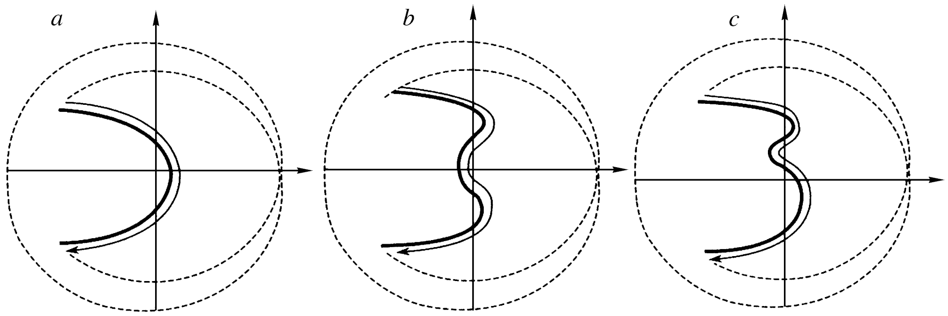

Fig. 5. Contour l in the plane of the function $A(c)+B(c)+G(c)$

We will consider two cases. Let the segments $[-\chi;\ \chi]$ and $[\mathrm{M}-1;\ \mathrm{M}+1]$ intersect. Then the image of the curve l under the action of the left side of (2.1) looks as shown in Fig. 2. It makes one turn around zero and, hence, Eq. (2.1) has complex roots. The l image under the action of the function specified by the left side of (1.3) then lies in an arbitrarily small (as $k \to 0$) neighborhood of the curve depicted in Fig. 2 and, hence, the dispersion equation (1.3) also has complex roots. Thus, in this case the system is unstable.

Now let the segments $[-\chi;\ \chi]$ and $[\mathrm{M}-1;\ \mathrm{M}+1]$ not intersect. Then only the image of the segment $[\chi;\ \mathrm{M}-1]$ can lie in the neighborhood of zero. In this case, Eq. (2.1) may or may not have complex roots. In general, if the plate is taken into account, for small k the first term in (1.3) always ensures the existence of complex roots. Let us prove this.

In Section 2 it was shown that the function $A(c)+B(c)$ specified by the left side of (2.1) maps the segment $[\chi,\ \mathrm{M}-1]$ into a segment of the imaginary axis, χ being mapped into a point with positive imaginary part and $\mathrm{M}-1$ into a point with negative imaginary part. The image of the entire segment $[\chi;\ \mathrm{M}-1]$ can pass through zero 1 or 3 times.

Let us consider the function

$$G(c)=k(c^2-\mathrm{M}_w^2-Dk^2)\sqrt{\chi^2-c^2}\sqrt{1-(c-\mathrm{M})^2}$$

where k is sufficiently small. On the interval $(\chi;\ \mathrm{M}-1)$ the product of two square roots in this function is positive; therefore, the total function may, depending on the $Dk^2+\mathrm{M}_w^2$ value, be positive, or negative, or change sign once from “minus” to “plus”. In the first two cases, the image of the curve l under the action of $A(c)+B(c)+G(c)$ looks as shown in Fig. 5*a* and *b*. In these cases, the image turns once and twice, respectively, around zero and the dispersion equation (1.3) has complex roots.

We will now assume that the function G(c) changes sign. We will denote the point at which $G(c)=0$ by c_0. Then $G(c)<0$ for $\chi<c<c_0$ and $G(c)>0$ for $c_0<c<\mathrm{M}-1$. Since on the segment $[\chi;\ \mathrm{M}-1]$ the quantity $A(c)+B(c)$ has purely imaginary values, we obtain:

$$\mathrm{Re}(A(c)+B(c)+G(c))<0, \quad c<c_0 \qquad \mathrm{Re}(A(c)+B(c)+G(c))>0, \quad c>c_0$$

Depending on the sign of $\mathrm{Im}(A(c_0)+B(c_0))=\mathrm{Im}(A(c_0)+B(c_0)+G(c_0))$, the image of l looks as shown in Fig. 5*b* or *c*. It can be seen that the image of the curve l makes one or two turns around zero and the dispersion equation (1.3) has complex roots.

Thus, for small k and arbitrary values of the parameters M, M_w, D, κ, and χ, the sinusoidal perturbations grow with time, that is, the system is unstable.

Similarly, we can prove the instability of the system in which both the upper and lower gases move at different arbitrary velocities. The case of a single gas moving above and below the plate at identical velocities can be reduced to the case of unilateral flow past a plate at a constant pressure on the opposite side.

We will now estimate the growth rate and number of the growing perturbations and the stability effect of the plate for various k when $D\neq 0$ (when $D=0$ this analysis can be similarly performed).

Let $\mathrm{M} - 1 < \chi$. Then for a tangential discontinuity, at any k, there is a single perturbation with $\operatorname{Im}\omega > 0$. For a plate, at small k, there is also a single growing perturbation, whose frequency, as $k \to 0$, approaches the perturbation frequency for the tangential discontinuity. With increase in k, on the intersection of the segments $[-\chi;\ \chi]$ and $[\mathrm{M} - 1;\ \mathrm{M} + 1]$ the function $G(c)$ becomes negative and "carries away" the contour "bill" in Fig. 2 into the left half-plane. This means that the root ω with positive imaginary part becomes real and, thus, at large k the plate has a stabilizing effect.

Now let $\mathrm{M} - 1 > \chi$. In this case, the tangential discontinuity can have one unstable perturbation or no unstable perturbations at all. For small k, if $c^2 - \mathrm{M}_w^2 > 0$ on the segment $[\chi;\ \mathrm{M} - 1]$ (that is, $\mathrm{M}_w < \chi$), the plate has one unstable perturbation (Fig. 5*a*). If $c^2 - \mathrm{M}_w^2 \le 0$ on this segment (that is, $\mathrm{M} - 1 \le \mathrm{M}_w$), there are two unstable perturbations. With increase in k the contour is displaced into the left half-plane (Fig. 5*b*) and another unstable perturbation appears.

Similarly, we can show that for $\mathrm{M} - 1 < \chi$, with increase in the plate parameters D and M_w, each perturbation with a fixed k becomes stable, that is, the plate has a stabilizing effect. On the other hand, if $\mathrm{M} - 1 > \chi$, for each fixed k two growing perturbations appear.

4. ANALYSIS OF THE PLATE PROBLEM FOR ZERO DENSITY OF THE REST GAS

We will now consider the case of a plate unilaterally flowed over by a gas, while on the other side a substance with a constant pressure equilibrating the plate and zero density is situated. This formulation is considered in the great majority of publications on the stability of plates in a gas flow [1, 2, 5, 6]. The dispersion equation of this problem can be written in the form:

$$k(c^2 - \mathrm{M}_w^2 - Dk^2)\sqrt{1 - (c - \mathrm{M})^2} + (c - \mathrm{M})^2 = 0 \tag{4.1}$$

We will show that in this case the system is stable if and only if $\mathrm{M} \le \mathrm{M}_w$.

First let $\mathrm{M} > \mathrm{M}_w$. We make the substitution in (4.1): $c = \mathrm{M} + isk^{1/2}$, where s is the new variable

$$s^2 = ((isk^{1/2} + \mathrm{M})^2 - \mathrm{M}_w^2 - Dk^2)\sqrt{1 + ks^2}$$

We can see that $s \to \pm\sqrt{\mathrm{M}^2 - \mathrm{M}_w^2}$ as $k \to 0$. Hence, the asymptotic behavior of c is given by the number $\mathrm{M} + i\sqrt{\mathrm{M}^2 - \mathrm{M}_w^2}\,k^{1/2}$ and the system is unstable.

Now let $\mathrm{M} \le \mathrm{M}_w$. Then, as in Section 2, for any k the image of the curve l mapped by the function determined by the left side of (4.1) does not turn around zero. Using the argument principle, we conclude that (4.1) has no complex roots and the system is stable.

The condition $\mathrm{M} \le \mathrm{M}_w$ obtained is the criterion of long-wave stability. If instability takes place, each k corresponds to a single growing perturbation. For large k, the perturbation is stable only if $\mathrm{M} \le \sqrt{\mathrm{M}_w^2 + Dk^2} + 1$. If $D \ne 0$, then as $k \to \infty$, all the perturbations are stable, whereas if $D = 0$, for $\mathrm{M} \le \mathrm{M}_w + 1$ the short waves are stable and for $\mathrm{M} > \mathrm{M}_w + 1$ all the perturbations are unstable together with the short waves. The proof of this is too cumbersome to be reproduced here. It can be seen from these criteria that with increase in the parameter M_w (at least up to $\mathrm{M}_w = \mathrm{M}$) any preassigned perturbation becomes unstable. On the other hand, if $\mathrm{M} > \mathrm{M}_w$, then with increase in the parameter D the short waves become stable, while the long waves, although they continue to grow, grow more slowly.

For perturbations oriented at an angle to the gas flow the form of the dispersion equation does not change, only M decreases. Therefore, for plane sinusoidal perturbations with an arbitrary wave vector the stability criterion coincides with that obtained above.

The stability analysis for a plate flowed past on both sides by the same gas at identical velocities can be reduced to the case considered. For this problem the stability condition coincides with the criterion $\mathrm{M} \le \mathrm{M}_w$ obtained above.

Summary. The stability of a tangential discontinuity between two gases and two types of flow past a plane plate is analyzed. The stability criterion contains five dimensionless parameters characterizing the system:

M — the Mach number of the flowing gas, M_w — the ratio of the propagation velocity of small long-wave perturbations in the plate to the speed of sound in the moving gas, D — the dimensionless flexural stiffness of the plate, and κ and χ — the rest to moving gas density and sonic speed ratios.

For a tangential discontinuity, on the assumption that the speed of sound is lower in the rest than in the moving gas, the stability region in the space of the parameters (M, χ, κ) is found qualitatively. For given gases there exists a single critical Mach number, such that the tangential discontinuity is stable at higher and unstable at lower Mach numbers. A two-sided estimate of the critical number, that is, a sufficient condition of tangential discontinuity stability/instability, is obtained.

The system consisting of a plate with a gas flowing on one side and at rest on the other is unstable for any problem parameters: long waves always grow. The behavior of the other waves depends on the Mach number. If $M < \chi + 1$, then for $D \neq 0$ short-wave perturbations are stable. In this case, the plate has a stabilizing effect: with increase in the parameters M_w and D an arbitrary perturbation with given wavelength becomes stable, although growing perturbations with sufficiently large wavelengths can be found. If $M > \chi + 1$, then for $D \neq 0$ the short waves grow. The effect of the plate is now destabilizing: with increase in M_w and D any perturbation with a given wavelength starts to grow.

The system consisting of a plate unilaterally flowed over by a gas, with a zero-density substance at constant pressure on the other side, is unstable only if $M > M_w$, in which case long waves grow. This is also valid for a plate bilaterally flowed over by the same gas at identical velocities on both sides.

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REFERENCES

1. A. A. Movchan, “On the oscillations of a plate moving in a gas,” *Prikl. Mat. Mekh.*, **20**, No. 2, 211–222 (1956).
2. A. A. Ilyushin and I. A. Kiiko, “Oscillations of a rectangular plate in a supersonic gas flow,” *Vestn. MGU, Ser. 1, Mathematics & Mechanics*, No. 4, 40–44 (1994).
3. A. A. Ilyushin, *Continuum Mechanics* [in Russian], Moscow University Press, Moscow (1990).
4. L. D. Landau and E. M. Lifshits, *Theoretical Physics. V. 6. Hydromechanics* [in Russian], Nauka, Moscow (1986).
5. M. Goland and Y. L. Luke, “An exact solution for two-dimensional linear panel flutter at supersonic speeds,” *J. Aeronaut. Sci.*, **21**, No. 4, 275–276 (1954).
6. V. V. Bolotin, *Nonconservative Problems of Elastic Stability Theory* [in Russian], Fizmatgiz, Moscow (1961).
7. A. G. Kulikovskii, “On the stability of homogeneous states,” *Prikl. Mat. Mekh.*, **30**, No. 1, 148–153 (1966).
8. L. Brevdo and A. Il’ichev, “Multi-modal destabilization of a floating ice layer by wind stress,” *Cold Reg. Sci. Technol.*, **33**, No. 1, 77–89 (2001).
9. R. A. Cairns, “The role of negative energy waves in some instabilities of parallel flows,” *J. Fluid Mech.*, **92**, No. 1, 1–14 (1979).
10. L. A. Ostrovskii, S. A. Rybak, and L. Sh. Tsimring, “Negative-energy waves in hydrodynamics,” *Usp. Fiz. Nauk*, **150**, No. 3, 417–437 (1986).
11. A. G. Kulikovskii and I. S. Shikina, “Effect of viscosity on the stability of a tangential discontinuity in an incompressible fluid,” *Vestn. MGU, Ser. 1, Mathematics & Mechanics*, No. 6, 29–32 (1997).
12. V. V. Alekseev, “On the form of stability loss of a boundary layer on a flexible surface at high Reynolds numbers,” *Prikl. Mat. Mekh.*, **66**, No. 5, 811–823 (2002).
13. V. I. Matyash, “Flutter of an elasticoviscous plate,” *Mekhanika Polimerov*, No. 6, 1077–1083 (1971).
14. G. S. Larionov, “Nonlinear flutter of an elasticoviscous plate,” *Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela*, No. 4, 95–100 (1974).
15. I. A. Kiiko, “Flutter of a viscoelastic plate,” *Prikl. Mat. Mekh.*, **60**, No. 1, 172–175 (1996).