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HIGH-FREQUENCY FLUTTER OF RECTANGULAR PLATES

Vasily V. Vedeneev

Department of Hydromechanics
Faculty of Mechanics and Mathematics
Lomonosov Moscow State University
E-mail: Vasily.Vedeneev@mail.ru

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Summary. *Stability of a rectangular elastic plate in a supersonic gas flow is analyzed. High-frequency disturbances, for which the piston theory cannot be used, are considered. Theory of global instability generalized to two-dimensional disturbances [5] is used. For each plate eigenmode the flutter criterion is derived. Mechanism of eigenmodes growth is described in detail.*

1 INTRODUCTION

In recent papers [1, 2] with use of the theory of global instability [3] flutter of plate having form of a strip in a supersonic gas flow was investigated. Two types of flutter were obtained. One of them is a classical panel flutter [4], which occurs due to interaction of two oscillation modes. Another one is a high-frequency flutter, which is the consequence of negative aerodynamic damping of one of the eigenmodes. This flutter type was derived for the first time: its essential feature is that it cannot be obtained with use of the piston theory. In the present paper investigation [1, 2] is generalized to rectangular plates. Stability of high-frequency disturbances is analyzed.

2 REDUCING THE PROBLEM TO RUNNING WAVES

Let us have thin elastic stretched rectangular plate, exposed to a supersonic homogeneous inviscid gas flow. Gas velocity vector is parallel to the plate plane, its direction in this plane is arbitrary. The problem is to investigate stability of the system.

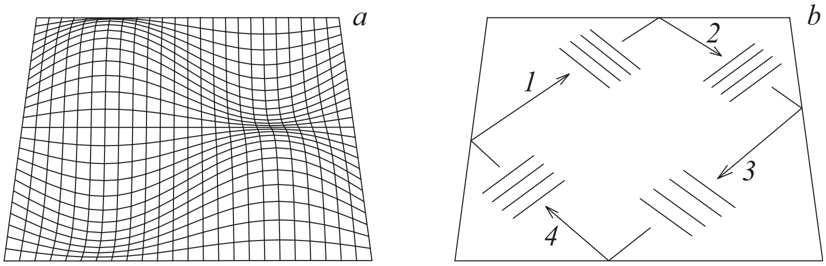

Figure 1. Plate eigenmode (a) and its representation as a superposition of four running waves (b)

Consider some natural mode of the plate. Suppose that plate length and width are sufficiently large, so that dynamic boundary effect is actual. Then the mode can be written in the form

$$\begin{aligned} w(x,y,t) &= \cos(k_x x + \varphi_x)\cos(k_y y + \varphi_y)e^{-i\omega t} = \\ &= \frac{1}{4}(e^{i\varphi_x}e^{ik_x x} + e^{-i\varphi_x}e^{-ik_x x})(e^{i\varphi_y}e^{ik_y y} + e^{-i\varphi_y}e^{-ik_y y})e^{-i\omega t} = \\ &= C_1 e^{i(k_x x + k_y y - \omega t)} + C_2 e^{i(k_x x - k_y y - \omega t)} + C_3 e^{i(-k_x x - k_y y - \omega t)} + C_4 e^{i(-k_x x + k_y y - \omega t)} \end{aligned} \qquad (1)$$

where $k_{x,y}$ and $\varphi_{x,y}$ are wave numbers and phases. Using this expression, represent the natural mode, which is a standing wave, as a superposition of running waves. Imagine that at one of the plate boundaries the running wave 1 is generated (fig. 1). After successive reflections from other boundaries, it transforms to waves 2, 3, 4 and after the last reflection — to initial wave 1. After that the process is repeated cyclically and finally leads to generation of the standing wave (1). Thus we have reduced the problem to running waves: if influence of the gas on these waves is known, it is simple to derive its action on the natural mode as a whole.

Action of the gas on running waves was investigated in [1, 2]. Considering motion of the plate-gas system in the form $e^{i(k_x x + k_y y - \omega t)}$ and substituting it in the system of equation, we finally obtain dispersion relation, which is connection between wave vector $\vec{k} = \{k_x; k_y\}$ and frequency ω:

$$(Dk^4 + M_w^2 k^2 - \omega^2) - \mu \frac{(\omega - Mk\cos\alpha)^2}{\sqrt{k^2 - (\omega - Mk\cos\alpha)^2}} = 0 \qquad (2)$$

Here $k = \sqrt{k_x^2 + k_y^2}$ is the length of the wave vector and α is the angle between the wave vector and gas flow velocity vector. Dimensionless parameters in the dispersion equation are: D is the dimensionless plate stiffness, M_w is the plate tension, M is the Mach number and μ is the ratio of the gas density to density of the plate material. Assuming $\mu<<1$, $k>> \mu$, $\omega>> \mu$, from (2) we obtain: $k=k_0+\Delta(k_0)$, where k_0 is length of the wave vector at vacuum (μ=0) and $\Delta(k_0) \sim \mu$. Growth or damping of the wave depends on the sign of imaginary part of $\Delta(k_0)$: if $\mathrm{Im}\Delta(k_0)<0$ then the wave amplifies, if $\mathrm{Im}\Delta(k_0)>0$ then it damps. Consequently, as for every eigenmode we know the wave vectors of all four running waves (1), from (2) we obtain increments (or decrements) of these waves in the presence of the gas.

3 CALCULATION OF AMPLIFICATION OF THE EIGENMODE

Consider trajectory of disturbance motion for one cycle of reflections (fig. 2). Such trajectory can be closed (fig. 2, a and b) or unclosed (fig. 2, c). Denoting l_j as summary length of the way passing in the j direction (fig. 1, b), noting that $l_1 = l_3$, $l_2 = l_4$, and using rule [5] for calculation of oscillation increment, we derive:

$$\delta = -\left.\frac{\partial\omega}{\partial k}\right|_{\mu=0} \frac{l_1 \,\mathrm{Im}(\Delta(k_1) + \Delta(k_3)) + l_2 \,\mathrm{Im}(\Delta(k_2) + \Delta(k_4))}{2(l_1 + l_2)},$$

where δ is the increment of the whole natural mode and k_j is the wave vector of the j direction. Inequality $\delta>0$ is the high-frequency flutter criterion of the plate natural mode.

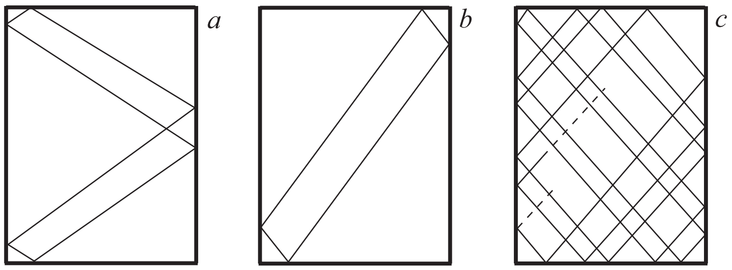

Figure 2. Trajectories of the disturbance propagation: closed (a, b) and unclosed (c).

Using this condition and considering different trajectories, which depend on the problem parameters and the natural mode, we find influence of these parameters on flutter generation, growth speed and other properties.

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REFERENCES

[1] Vedeneev, V.V. (2005): Flutter of a wide strip plate in a supersonic gas flow, *Fluid Dynamics*, Vol. 40, № 5, pp. 805-817.

[2] Vedeneev, V.V. (2005): Analytical investigation of panel flutter in supersonic gas flow, *European Conference for Aerospace Sciences (EUCASS), Moscow, Russia, 4-7 July*.

[3] Kulikovskii, A.G. (1966): On the stability of homogeneous states, *Journal of Applied Mathematics and Mechanics*, Vol. 30, Issue 1, pp. 180-187.

[4] Dowell, E.H. (1970): Panel flutter: a review of the aeroelastic stability of plates and shells. *AIAA Journal,* Vol. 8, № 3, pp. 385-399.

[5] Kulikovskii, A.G. (2006): On global instability of uniform flows in non-one dimensional domains, *Journal of Applied Mathematics and Mechanics*, Vol. 70, Issue 2 (in press).