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High-Frequency Flutter of a Rectangular Plate

V. V. Vedeneev

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Abstract — In our recent study of strip plate flutter, two flutter types, low- and high-frequency, were revealed. The first arises due to interaction of the oscillation modes and has been investigated in detail in numerous studies using an approximate piston theory. The high-frequency flutter, detected for the first time, is a consequence of the presence of negative aerodynamic damping and cannot be obtained using the piston theory. In the present study, the high-frequency flutter of a rectangular plate is investigated.

Keywords: high-frequency flutter, plate flutter, panel flutter, plate stability, global instability.

In [1, 2], using the method of global instability of one-dimensional systems [3, 4, §65], the stability of a strip plate in a supersonic gas flow was investigated. In [5], this method was extended to the case of two-dimensional systems. In this study, it is applied to the stability of a rectangular plate.

As applied to the problem in question, the principle of this method is as follows. For a sufficiently large plate, each natural oscillation can be represented as a superposition of four traveling waves. The effect of the gas on these waves was investigated in [1, 2]. Using the results of those studies, it is possible to calculate the relative change in amplitude after successive wave reflections. Comparing this change with unity, we can determine whether the oscillations are damped or amplified under the action of the gas.

1. FORMULATION OF THE PROBLEM

A thin elastic, isotropically stretched rectangular plate is exposed on one side to a uniform supersonic inviscid perfect gas flow, while on the other side a constant pressure equal to the unperturbed gas pressure is maintained. The plate is embedded in an absolutely rigid plane separating the flow from the constant pressure region. It is required to investigate the system stability with respect to high-frequency perturbations when the gas has only a slight effect on the plate oscillations.

We introduce the coordinate system xyz as shown in Fig. 1. The gas flows in the region $z > 0$ at an angle θ to the x axis, and the plate occupies the region W: $|x| < L_x/2$, $|y| < L_y/2$, $z = 0$. The width and length of the plate L_x and L_y are assumed to have been nondimensionalized by dividing by its thickness. In addition, the problem also contains the following dimensionless parameters:

$$\mathrm{M} = \frac{u}{a}, \qquad \mathrm{M}_w = \frac{\sqrt{\sigma/\rho_m}}{a}, \qquad D = \frac{D_w}{a^2\rho_m h^3}, \qquad \mu = \frac{\rho}{\rho_m}$$

The quantities u, a, and ρ are the flow velocity, the speed of sound, and the gas density, σ is the tensile stress in the mid-plane of the plate, ρ_m and h are the material density and thickness of the plate, and $D_w = Eh^3/(12(1-\nu^2))$ is its flexural stiffness. We will assume that $\mathrm{M} > 1$ and $\mu \ll 1$. We will also assume that the inequalities $L_x \gg 1$ and $L_y \gg 1$ hold. The meaning of these relations will be refined below.

The differential equations and boundary conditions have the following form:

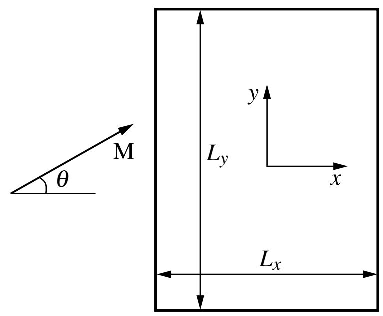

Fig. 1. Configuration of the system considered

$$\left(\frac{\partial}{\partial t} + \mathrm{M}\cos\theta\frac{\partial}{\partial x} + \mathrm{M}\sin\theta\frac{\partial}{\partial y}\right)^2\varphi - \frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial y^2} - \frac{\partial^2\varphi}{\partial z^2} = 0, \quad z > 0$$

$$\frac{\partial\varphi}{\partial z} = \frac{\partial w}{\partial t} + \mathrm{M}\cos\theta\frac{\partial w}{\partial x} + \mathrm{M}\sin\theta\frac{\partial w}{\partial y}, \quad z = 0, \quad (x,\ y) \in W$$

$$\frac{\partial\varphi}{\partial z} = 0, \quad z = 0, \quad (x,\ y) \notin W$$

$$\frac{\partial\varphi}{\partial q} = 0, \qquad z \to +\infty, \qquad q = x,\ z,\ t$$

$$\frac{\partial^2 w}{\partial t^2} = \mu\left(\frac{\partial\varphi}{\partial t} + \mathrm{M}\cos\theta\frac{\partial\varphi}{\partial x} + \mathrm{M}\sin\theta\frac{\partial\varphi}{\partial y}\right) + \mathrm{M}_w^2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) -$$

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4}\right), \qquad z = 0, \qquad (x,\ y) \in W$$

$$Fw = 0, \qquad z = 0, \qquad (x,\ y) \in \partial W$$

Here, φ and w are the dimensionless gas perturbation potential and plate deflection. The first equation is the wave equation, the second and third are the impermeability conditions, the fourth is the condition of perturbation damping in the gas far from the plate, the fifth is the equation of plate motion, and the sixth represents the boundary conditions on the plate edges (F is a differential operator that assigns the boundary conditions on each edge).

2. CONDITION OF PLATE OSCILLATION AMPLIFICATION

Let us consider the vacuum mode of plate oscillation which can be characterized by the numbers of half-waves m and n in the x and y directions, respectively. We will assume that the plate is sufficiently large for the dynamic boundary effect [6, §34] to take place, that is, far from the plate edges, irrespective of the boundary conditions, any oscillation shape can be represented by the expression

$$w(x,\ y,\ t) = \cos(k_x x + \varphi_x)\cos(k_y y + \varphi_y)e^{-i\omega t}$$

Here, $\omega \in \mathbf{R}$ is the oscillation frequency and k_x, k_y, φ_x, and φ_y are the wave numbers and phase shifts which depend on the numbers of half-waves m and n, the boundary conditions, and the plate dimensions and properties. In particular, for all edges hinged, they have the simple form:

$$k_x = \frac{m\pi}{L_x}, \qquad \varphi_x = \frac{(m-1)\pi}{2}, \qquad k_y = \frac{n\pi}{L_y}, \qquad \varphi_y = \frac{(n-1)\pi}{2}$$

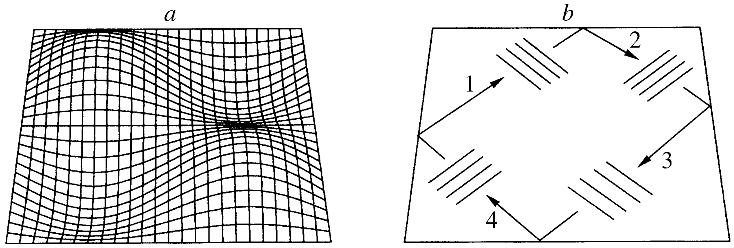

Fig. 2. Natural shape of plate oscillations — standing wave (*a*) and its representation in the form of a superposition of four traveling waves (*b*). The directions of wave propagation are denoted by the numbers

We will represent the oscillation shape as a superposition of four traveling waves:

$$w(x,\ y,\ t) = \frac{1}{4}(e^{i\varphi_x}e^{ik_x x} + e^{-i\varphi_x}e^{-ik_x x})(e^{i\varphi_y}e^{ik_y y} + e^{-i\varphi_y}e^{-ik_y y})e^{-i\omega t} =$$
$$C_1 e^{i(k_x x + k_y y - \omega t)} + C_2 e^{i(k_x x - k_y y - \omega t)} + C_3 e^{i(-k_x x - k_y y - \omega t)} + C_4 e^{i(-k_x x + k_y y - \omega t)} \quad (2.1)$$

We will number the wave propagation directions in accordance with the number of the terms in (2.1) (Fig. 2). Then the formation of a standing wave can be represented as follows. On one edge a traveling wave is excited and then moves, for example, in direction 1. On successive reflection from the four plate edges, it transforms into waves that move in directions 2, 3, and 4. After the last reflection it transforms into the initial wave, after which the cycle is repeated. After several cycles the motion of these four waves becomes stable and their superposition leads to the formation of a standing wave.

Let the plate now be placed in a gas flow. We will neglect the effect of the plate edges on the flow perturbation and assume that the flow acts on the traveling waves (2.1) as if they were of infinite amplitude. If the effect of the gas on such waves is known, it is easy to understand its effect on the natural oscillations as a whole. Below, the main results of our investigation of the action of a flow on waves of infinite amplitude [1, 2] are presented.

We will consider a plane wave $e^{i(k_x x + k_y y - \omega t)}$ with a wave vector $\{k_x,\ k_y\} = \{k\cos(\alpha + \theta),\ k\sin(\alpha + \theta)\}$, which travels along a fictitious unbounded plate in a gas flow. Here, α is the angle between the gas velocity vector and the wave vector, $k = \sqrt{k_x^2 + k_y^2}$, and $\mathrm{Re}\,k > 0$. The wave number k is real and positive in the absence of a gas and generally complex if a gas is present. The wave is damped in its direction of propagation if $\mathrm{Im}\,k$ is positive and amplified if it is negative.

The dispersion equation of the plate-gas system has the form:

$$(Dk^4 + \mathrm{M}_w^2 k^2 - \omega^2) - \mu \frac{(\omega - \mathrm{M}k\cos\alpha)^2}{\sqrt{k^2 - (\omega - \mathrm{M}k\cos\alpha)^2}} = 0 \quad (2.2)$$

From (2.2), denoting the wave vector length in the absence of a gas ($\mu = 0$) by k_0 and assuming μ to be a small parameter, we obtain $k = k_0 + \Delta(k_0)$, where $\Delta(k_0)$ is the leading term of the $k(\mu)$ expansion for small μ. If the phase velocity of the wave $c_0 = \omega/k_0$ lies outside the small neighborhood of $\mathrm{M}\cos\alpha \pm 1$, then

$$\Delta(k_0) = \frac{\mu(\omega - \mathrm{M}k_0\cos\alpha)^2}{2k_0(2Dk_0^2 + 2\mathrm{M}_w^2)\sqrt{k_0^2 - (\omega - \mathrm{M}k_0\cos\alpha)^2}} \quad (2.3)$$

If c_0 lies in the neighborhood of $\mathrm{M}\cos\alpha - 1$, then the imaginary part of the increment $\Delta(k_0)$ is negative and of the order of $\mu^{2/3}$. If $c_0 = \mathrm{M}\cos\alpha - 1$, the value $\mathrm{Im}\,\Delta(k_0)$ reaches an absolute minimum and

$$\Delta(k_0) = \mu^{2/3}\left(\frac{\sqrt{(\mathrm{M}-1)^2 - \mathrm{M}_w^2}}{8\sqrt{D}(\mathrm{M}-1)^3}\left(\frac{2(\mathrm{M}-1)^2 - \mathrm{M}_w^2}{\mathrm{M}-1}\right)^{-2}\right)^{1/3} e^{-i\pi/3} \quad (2.4)$$

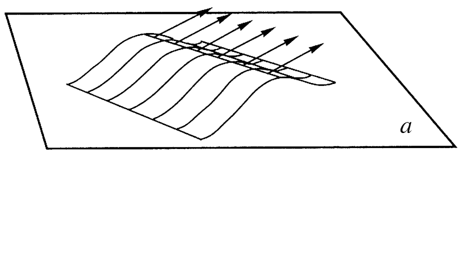

Fig. 3. Representation of the wave motion as the motion of its individual segments (*a*) and the initial trajectories of these segments (*b*)

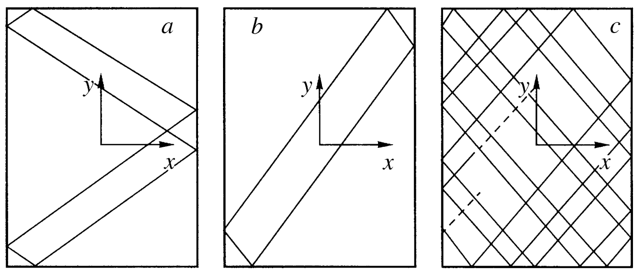

Fig. 4. Wave segment trajectories: closed trajectory symmetrical about one of the coordinate axes (*a*), closed trajectory asymmetrical about the coordinate axes (*b*), open trajectory (*c*)

In the neighborhood of $\mathrm{M}\cos\alpha+1$, the quantity $\mathrm{Im}\Delta(k_0)$ is positive and also of the order of $\mu^{2/3}$; at $c_0=\mathrm{M}\cos\alpha\,+\,1$ it reaches an absolute maximum.

The physical significance of these relations is as follows. The gas velocity vector can be decomposed into two components: one in the wave plane and the other perpendicular to it. In the direction perpendicular to the wave plane the gas velocity is equal to $\mathrm{M}\sin\alpha$ and the flow does not affect the wave. In the wave plane the velocity is equal to $\mathrm{M}\cos\alpha$ and it is this component that determines the action of the gas on the wave traveling along the plate. If $c_0<\mathrm{M}\cos\alpha\,-\,1$, the gas velocity relative to the wave is supersonic and the wave motion direction coincides with the direction of gas flow relative to the wave. If $\mathrm{M}\cos\alpha\,-\,1< c_0<\mathrm{M}\cos\alpha\,+\,1$, the gas flow relative to the wave is subsonic. If $c_0>\mathrm{M}\cos\alpha\,+\,1$, the gas flow relative to the wave is again supersonic but the motion of the wave is opposite in direction to the gas flow relative to it. Taking into account the phase shift between the wave traveling along the plate and the pressure acting on the wave and calculating the work done by the pressure per wave period, we obtain [1]

$$c_0<\mathrm{M}\cos\alpha\,-\,1 \quad\Longrightarrow\quad \mathrm{Im}\Delta(k_0)<0,$$

$$\mathrm{M}\cos\alpha\,-\,1<c_0<\mathrm{M}\cos\alpha\,+\,1 \quad\Longrightarrow\quad \mathrm{Im}\Delta(k_0)=0,$$

$$c_0>\mathrm{M}\cos\alpha\,+\,1 \quad\Longrightarrow\quad \mathrm{Im}\Delta(k_0)>0$$

If $c_0=\mathrm{M}\cos\alpha\pm 1$, resonance occurs between the waves propagating in the gas at a velocity $\mathrm{M}\cos\alpha\pm 1$ (leading and trailing fronts of the acoustic perturbation) and along the plate at a velocity c_0. This resonance is also the cause of the maximum amplification and damping of the wave at these c_0 values.

We will now consider the action of the gas on the natural oscillations. For this purpose it is convenient to represent the motion of the wave as a motion of its separate segments (Fig. 3). Between the edges the trajectories of these segments are rectilinear with one of the four directions (Fig. 2*b*), and from the edges mirror reflection occurs. Depending on the oscillation mode considered, the trajectories may be closed (Fig. 4*a* and *b*) or open (Fig. 4*c*). The closed trajectory is a closed broken line. It is easy to show that the open trajectory is everywhere dense in the rectangle defined by the plate contour.

We will call the period in which the wave segment trajectories return to their initial position (if the trajectory is closed) or close to it (if open) the wave segment reflection cycle. By calculating along each trajectory the change in amplitude during this cycle, we also find the change in amplitude for the wave as a whole, since the trajectories cover the entire plate surface. If on different trajectories the amplitude amplification is different, the wave ceases to be plane. In this case, diffraction effects may be manifested, but we will not take these into account.

We note that generally the wave segment reflection cycle does not coincide with the reflection cycle of the wave as a whole shown in Fig. 2*b*, which always consists of four reflections. In fact, after four reflections a wave segment may be reflected into another segment, not into itself, and the reflections of such segments do not form a closed cycle.

We will consider a certain trajectory and calculate the change in amplitude per reflection cycle. The amplitude changes, firstly, during wave motion from one edge to another due to the presence of the imaginary part of the wave number (motion along trajectory segments) and, secondly, during reflection from the plate edges. Considering successively the changes in amplitude during motion between the edges and during reflection and assuming that the initial amplitude is equal to unity and that in this time n reflections occur, we find the amplitude after the reflection cycle:

$$\prod_{p=1}^{n} A_p e^{-l_1 \operatorname{Im}\Delta(k_1) - l_2 \operatorname{Im}\Delta(k_2) - l_3 \operatorname{Im}\Delta(k_3) - l_4 \operatorname{Im}\Delta(k_4)}$$

Here, A_i are the coefficients of reflection from the edges and l_j are the total distances traversed by the trajectory in the j-direction. Since in the absence of a gas the amplitude does not change after the reflection cycle (the plate itself is a conservative system):

$$\prod_{p=1}^{n} A_p = 1$$

It is also obvious that $l_1 = l_3$ and $l_2 = l_4$ (in the case of an open trajectory these equalities hold approximately true). Therefore, the oscillation amplification condition takes the final form:

$$l_1 \operatorname{Im}(\Delta(k_1) + \Delta(k_3)) + l_2 \operatorname{Im}(\Delta(k_2) + \Delta(k_4)) < 0 \tag{2.5}$$

Thus, condition (2.5) makes it possible to find out whether the amplitude increases or not on each trajectory of a fixed oscillation mode. If on all the trajectories the amplitude increases, the oscillation itself will be amplified and if on all the trajectories it decreases, the oscillation will be damped. If on certain trajectories the amplitude increases and on other trajectories it decreases, the net behavior will be determined by wave diffraction which can be taken into account by solving the exact oscillation equation for a plate in a gas flow.

The growth increment can be found from the formula [5]

$$\delta = -g(\omega)\frac{l_1 \operatorname{Im}(\Delta(k_1) + \Delta(k_3)) + l_2 \operatorname{Im}(\Delta(k_2) + \Delta(k_4))}{2(l_1 + l_2)} \tag{2.6}$$

where $g(\omega) = (\partial k_0/\partial\omega)^{-1}$ is the group velocity of the waves.

3. CASE IN WHICH THE GAS VELOCITY VECTOR IS PARALLEL TO ONE OF THE PLATE SIDES

For definiteness, we will assume that the flow is directed along the x axis ($\theta = 0$). In this case, due to the symmetry, $\Delta(k_1) = \Delta(k_2)$ and $\Delta(k_3) = \Delta(k_4)$. Then the condition (2.5) of wave amplitude growth reduces to the condition

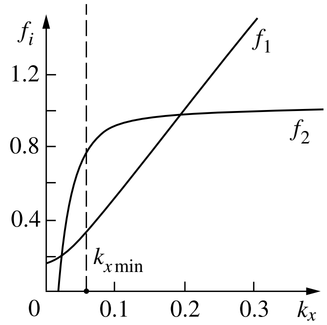

Fig. 5. Graphs of the functions $f_{1,2}(k_x)$ for the parameter values M = 2, $D = 23.9$, $M_w = 0$, and $k_{y\min} = 0.031$. These values correspond to a steel hinged plate in an air flow under normal conditions with the restrictions $L_x < 50$ and $L_y < 100$

$$\operatorname{Im}\Delta(k_1) < -\operatorname{Im}\Delta(k_3) \tag{3.1}$$

which expresses the fact that the amplification of waves 1 and 2 propagating downstream must exceed the damping of waves 3 and 4 propagating upstream. Since for the selected oscillation mode the trajectories differ from each other only with respect to the initial point and coincide in direction of motion, the change in amplitude after a reflection cycle and condition (3.1) are independent of the trajectory and are determined only by the oscillation mode.

Let us consider the effect of the parameters on the plate flutter.

Let M, M_w, D, and μ be given. We will find the condition for any plate (of any dimensions) to lie in the stability region. This will obviously be so if and only if $c_0 > \mathrm{M}\cos\alpha - 1$ for any k_x and k_y. Taking into account that from (2.1) and (2.2) $c_0 = \sqrt{D(k_x^2 + k_y^2) + \mathrm{M}_w^2}$ and $\cos\alpha = k_x/\sqrt{k_x^2 + k_y^2}$, we can write this inequality in the form:

$$\sqrt{D(k_x^2 + k_y^2) + \mathrm{M}_w^2} > \mathrm{M}\frac{k_x}{\sqrt{k_x^2 + k_y^2}} - 1 \tag{3.2}$$

Since the permissible values of k_x and k_y are arbitrary, we obtain the required range of parameter values:

$$\mathrm{M}_w > \mathrm{M} - 1$$

Let us consider the same problem on condition that the permissible plate dimensions L_x and L_y are bounded from above. In this case the permissible values of k_x and k_y are bounded from below. Obviously, in inequality (3.2) it is sufficient to consider $k_y = k_{y\min}$. We will consider the functions

$$f_1(k_x) = \sqrt{D(k_x^2 + k_{y\min}^2) + \mathrm{M}_w^2}, \qquad f_2(k_x) = \mathrm{M}\frac{k_x}{\sqrt{k_x^2 + k_{y\min}^2}} - 1$$

Both functions increase monotonically with k_x, f_1 being convex downwards and f_2 convex upwards and bounded by the value M − 1. Graphs of these functions are plotted in Fig. 5. The required parameter value range is determined by the following conditions: the graphs of the functions f_1 and f_2 either do not cross or cross with both intersection points lying to the left of the value $k_{x\min}$. In particular, it can be seen that for the parameter values corresponding to Fig. 5 we can find a plate that satisfies the given restrictions on size and lies in the flutter region.

We will note one property of the fastest-growing plate oscillation mode. If the flow is parallel to the x axis, expression (2.6) for the growth increment takes the form:

$$\delta = -\frac{1}{2}\left(\frac{\partial k_0}{\partial \omega}\right)^{-1} \operatorname{Im}(\Delta(k_1) + \Delta(k_3)) \tag{3.3}$$

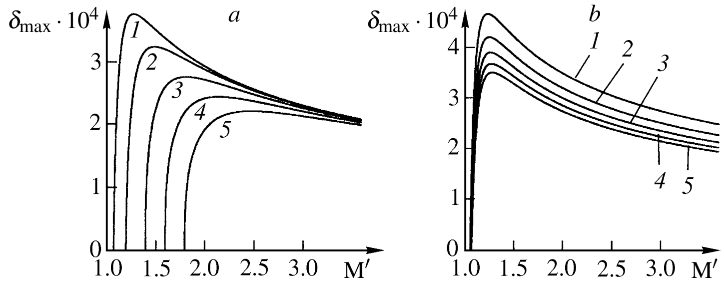

Fig. 6. Graphs of the functions $\delta_{\max}(\mathrm{M}')$ for $\mu = 1.2 \cdot 10^{-4}$; *a*: $D = 23.9$, $\mathrm{M}_w = 0,\ 0.2,\ 0.4,\ 0.6,\ 0.8$ (curves *1–5*); *b*: $\mathrm{M}_w = 0$, $D = 10,\ 15,\ 20,\ 25,\ 30$ (curves *1–5*)

The eigenfunctions that grow most rapidly are those for which the phase velocity of the downstream-traveling waves is $c_0 = \mathrm{M}\cos\alpha - 1$. Substituting expressions (2.3) and (2.4) for $\Delta(k_3)$ and $\Delta(k_1)$ in (3.3), we obtain

$$\delta_{\max}(\mathrm{M},\ \mathrm{M}_w,\ D,\ \mu,\ \alpha) = \delta_{\max}(\mathrm{M}',\ \mathrm{M}_w,\ D,\ \mu) =$$

$$\mu^{2/3}\frac{\sqrt{3}}{8}\left(\frac{(\mathrm{M}'-1)^2-\mathrm{M}_w^2}{D}\right)^{1/6}\frac{\left(2(\mathrm{M}'-1)^2-\mathrm{M}_w^2\right)^{1/3}}{(\mathrm{M}'-1)^{4/3}} - \mu\frac{(2\mathrm{M}'-1)^2}{4(\mathrm{M}'-1)\sqrt{(2\mathrm{M}'-1)^2-1}}$$

where $\mathrm{M}' = \mathrm{M}\cos\alpha$ and α is the positive acute angle between the gas velocity vector and the wave vectors of the downstream-traveling waves.

We will denote by $\delta^*_{\max}$ the greatest increment attainable on the eigenfunctions for fixed M, D, M_w, and μ and variable L_x and L_y. Obviously, $\delta^*_{\max} = \max\limits_{\alpha}\delta_{\max}(\mathrm{M}',\ \mathrm{M}_w,\ D,\ \mu)$. The graph of $\delta_{\max}(\mathrm{M}')$ is shown for several characteristic values of M_w, D, and μ in Fig. 6. This function has a single maximum at $\mathrm{M}' = \mathrm{M}_{\max}(\mathrm{M}_w,\ D,\ \mu)$. Then, if in the incident flow the number M is less than $\mathrm{M}_{\max}$, we have $\delta^*_{\max} = \delta_{\max}(\mathrm{M}',\ \mathrm{M}_w,\ D,\ \mu)$. The highest growth rate is displayed by the natural modes for which the angle α is least, that is, these modes have one half-wave in the y direction. If, on the other hand, $\mathrm{M} \geq \mathrm{M}_{\max}$, then $\delta^*_{\max} = \delta_{\max}(\mathrm{M}_{\max},\ \mathrm{M}_w,\ D,\ \mu)$, and we can select plate dimensions such that the maximum growth is displayed by the oscillation mode with several half-waves in the y direction. The most intense growth is displayed by the mode whose wave vector is directed at an angle $\alpha = \arccos(\mathrm{M}_{\max}/\mathrm{M})$ to the x axis and whose absolute value is such that $c_0 = \mathrm{M}\cos\alpha - 1 = \mathrm{M}_{\max} - 1$.

A similar property was noted in [1], where the problem of strip plate flutter was considered: for $\mathrm{M} > \mathrm{M}_{\max}$ plane perturbations have the maximum growth increment when the flow does not lie in the perturbation plane but is turned relative to it through an angle $\alpha = \arccos(\mathrm{M}_{\max}/\mathrm{M})$.

4. CASE IN WHICH THE GAS VELOCITY VECTOR IS NOT PARALLEL TO ONE OF THE PLATE SIDES

In this case, $\Delta(k_1) \neq \Delta(k_2)$ and $\Delta(k_3) \neq \Delta(k_4)$; therefore, for trajectories departing from different points on the plate in the same direction the amplitude amplification increments (2.6) are generally different, since the distances l_j are different. Because the direction in which a trajectory departs is determined by the oscillation mode, the free parameter that determines a particular trajectory is the point on the leading edge from which the trajectory departs.

All the trajectories can be divided into three groups: closed trajectories symmetrical about the x or y axis, closed trajectories asymmetrical about the x and y axes (in this case, they are symmetrical about the plate center), and open trajectories (Fig. 4).

We will first consider the case of closed symmetrical (to be specific, about the x axis) trajectories. For such trajectories we always have $l_1 = l_4$ and $l_2 = l_3$. Since for any closed trajectory $l_1 = l_3$ and $l_2 = l_4$, the distances traveled by a perturbation in all four directions are equal. Since the total length of the trajectory is independent of the initial point from which the trajectory departs, the perturbation amplification (2.6) is also independent of this point and the oscillation amplitude increases uniformly over the plate.

Let us now consider closed asymmetrical trajectories. It is easy to see that in this case for trajectories departing from different points the amplification coefficients will be different. The extremal trajectories depart from the corner points. There are two such trajectories connecting opposite corners of the plate. On one of these trajectories, which we will call the maximum trajectory, a perturbation is amplified the most and on the other the least. From this it follows that the plate oscillation amplitude is amplified nonuniformly over the plate: most rapidly at points close to the maximum trajectory. This effect is the more clearly expressed, the greater the difference between the distances l_1 and l_2, that is, the fewer the reflections per cycle. In particular, it is best expressed if the maximum trajectory is a diagonal of a rectangle (Fig. 4*b*); in this case, $l_2 = 0$. It is to be expected that this effect will be preserved in the nonlinear formulation: the amplitude of the steady-state plate oscillations will have a bulge around the maximum trajectory and will differ in this respect from the free oscillation shape of the plate in a vacuum.

We will now consider the case of open trajectories. After numerous reflections the distances traveled by the perturbation in all four directions will be approximately equal and the plate oscillations will be amplified uniformly over the plate, as in the case of closed symmetrical trajectories. If the trajectory direction is close to the direction of a certain closed asymmetrical trajectory, then during the first few reflection cycles the trajectories themselves will also remain close to one another and the growth will be maximal at points close to the maximum closed trajectory. As the number of reflections increases and the open trajectory moves away from the closed one, at other points of the plate the amplitudes will increase and gradually the growth of the plate oscillations will become uniform.

Summary. The stability of high-frequency perturbations of a rectangular plate in a supersonic gas flow is investigated. A condition that makes it possible to determine for each plate oscillation mode whether it is amplified or damped is obtained.

If the flow is parallel to one side of the plate, the oscillation is amplified or damped uniformly over the plate, that is, during flutter the oscillation shape remains undistorted. If the flow is not parallel to the edges and the oscillation mode corresponds to a closed asymmetrical perturbation propagation trajectory or to an open trajectory close to it, the oscillation shape changes: the amplitude is now greater near the maximum trajectory than at other points on the plate. The other oscillation modes remain undistorted under the action of the gas.

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