EUROMECH Colloquium 483
Geometrically Non-linear Vibrations of Structures
9 – 11 July 2007, FEUP, Porto, Portugal

NON-LINEAR ANALYSIS OF HIGH-FREQUENCY PANEL FLUTTER

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Keywords: panel flutter, plate flutter, non-linear flutter, high-frequency flutter, single-degree-of-freedom flutter.

1 INTRODUCTION

Recently a linear stability of an elastic plate in a supersonic gas flow was investigated using exact aerodynamic theory [1, 2]. Two types of instability were obtained. The first one is coupled-type instability: it happens due to interaction of two eigenmodes. This type is well described by the piston theory and analysed in linear and non-linear statements in detail [3, 4]. Another instability type is a single-degree-of-freedom instability. It was discovered for the first time in [1, 2] and was called "high-frequency flutter". Its main feature is that it cannot be detected using the piston theory. Simple physical mechanism of temporal eigenmodes amplification was discovered, and the criterion of linear stability was obtained. The aim of the present paper is to analyse high-frequency panel flutter in non-linear statement and to assess amplitudes of limit cycle oscillations.

2 STATEMENT OF THE PROBLEM

Von Karman's equation of non-linear plate motion in a gas flow in dimensionless variables has the following form (for simplicity, we consider two-dimensional problem):

$$D\frac{\partial^4 w}{\partial x^4}+\left(M_w^2+\frac{K}{2L}\int_{-L/2}^{L/2}\left(\frac{\partial w(\xi)}{\partial \xi}\right)^2 d\xi\right)\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial t^2}-P\{w\}=0\,.$$

Here D, M_w^2, K and L are dimensionless bending stiffness, static tension of the plate, non-linearity parameter and the plate length (the following dimensional parameters are used for non-dimensionalization: the plate thickness, the plate material density and the gas speed of sound). $P\{w\}$ is a pressure disturbance acting on the plate. If the plate oscillation is harmonic, then the exact theory of potential flow gives:

$$P\{W(x)\cos(\omega t)\}=-\mathrm{Re}\frac{\mu}{\sqrt{M^2-1}}e^{-i\omega t}\left(-i\omega+M\frac{\partial}{\partial x}\right)\int_{-L/2}^{x}\left(-i\omega W(\xi)+M\frac{\partial W(\xi)}{\partial \xi}\right)\times \exp\left(\frac{iM\omega(x-\xi)}{M^2-1}\right)J_0\left(\frac{-i\omega(x-\xi)}{M^2-1}\right)d\xi\,. \quad (1)$$

Here M is Mach number, μ is a ratio of the gas density to the plate material density. The same way as in [1, 2] we will assume that μ is a small parameter (in reality $\mu\sim 10^{-4}$). In case of non-harmonic oscillation one should express a deflection w as Fourier series or integral and use linearity of $P\{w\}$.

Let us assume that only the first mode ($w(x,t)=W_1(x)e^{-i\omega_1 t}$) is linearly unstable ($\mathrm{Im}\,\omega_1>0$). Then near the instability boundary a non-linear deflection has the form $w(x,t)\approx W_1(x)A_1(t)$. Using the procedure of Bubnov-Galerkin, we obtain the amplitude equation:

$$\frac{\partial^2 A_1}{\partial t^2} + \omega_{01}^2 A_1 + K a_{11}^2 A_1^3 - P\{A_1\} = 0 . \quad (2)$$

Here ω_{01} is the first plate eigenfrequency in vacuum. Expressions for a_{11}, $P\{A_1\}$ and the eigenform normalization condition are as follows:

$$a_{11} = \frac{1}{\sqrt{2L}} \int_{-L/2}^{L/2} \left(\frac{\partial W_1(x)}{\partial x} \right)^2 dx , \quad P\{A_1\} = \int_{-L/2}^{L/2} P\{W_1(x) A_1(t)\} W_1(x) dx , \quad \int_{-L/2}^{L/2} W_1^2(x) dx = 1 . \quad (3)$$

3 CALCULATION OF THE PRESSURE

As it is difficult to use (1) directly, we use the method based on the fact that we know eigenfrequency of the linearly unstable mode [1]. For harmonic oscillation ($A_1(t) = Ce^{-i\omega t}$) from (1) we have:

$$P\{A_1\} = p_1(\omega) A_1 + p_2(\omega) \frac{\partial A_1}{\partial t} , \quad (4)$$

where $p_1(\omega)$ and $p_2(\omega)$ are some functions of ω and of the problem parameters. If $\omega = \omega_{01}$, then substituting (4) into linearized equation (2), it is easy to see that $p_2(\omega_{01}) = \text{Im}\,\omega_1$. If $\omega \neq \omega_{01}$, we find such D' and M'^2_w that $\omega_{01}(D', M'^2_w) = \omega$ and $D'/D = M'^2_w / M^2_w$. Then we have: $p_2(\omega) = p_2(\omega_{01}(D', M'^2_w)) = \text{Im}(\omega_1(D', M'^2_w))$.

Finally, after calculations we obtain the dependence $p_2(\omega)$ shown in the figure 1. The frequencies ω', ω'' are functions of the problem parameters, and in particular of Mach number. Further we will need an expression for ω'; for example if the plate is simply supported and $M_w = 0$, then

$$\omega' = (M-1)\left(\frac{\pi}{L}\right), \qquad \omega_{01} = \sqrt{D}\left(\frac{\pi}{L}\right)^2 . \quad (5)$$

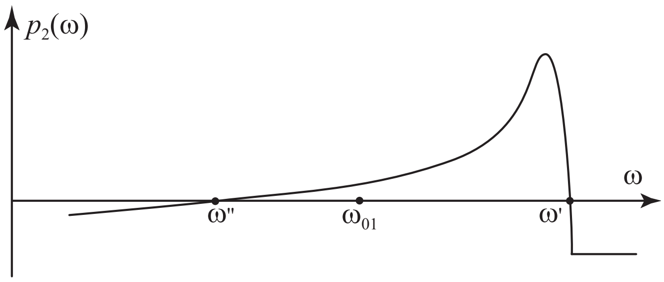

Figure 1. The function $p_2(\omega)$ in the high-frequency region.

In the region $\omega''(M) < \omega < \omega'(M)$ the aerodynamic damping ($-p_2(\omega)$) is negative; this is the reason of existence of a single-degree-of-freedom flutter. As $\omega'(M)$ and $\omega''(M)$ are growing functions of *M*, the first mode is unstable for $M^* < M < M^{**}$, where $\omega'(M^*) = \omega_{01}$ and $\omega''(M^{**}) = \omega_{01}$. Obviously, $M = M^*$ is a critical Mach number.

The method described above gives us the function $p_2(\omega)$, but says nothing about $p_1(\omega)$. On the other hand, as the high-frequency flutter mechanism is connected only with negative aerodynamic damping, and $p_1(\omega) \sim \mu << \omega_{01}^2$, then we may put $p_1(\omega) = 0$ with high accuracy.

4 LIMIT CYCLE ANALYSIS

Let us now return to the non-linear equation (2) where the expression (4) is used:

$$\frac{\partial^2 A_1}{\partial t^2} - p_2(\omega_1) \frac{\partial A_1}{\partial t} + \omega_{01}^2 A_1 + K a_{11}^2 A_1^3 = 0 . \quad (6)$$

As limit cycles are periodic solutions, we can search for them in the form

$$A_1(t) = C \cos \omega t + \sum_{n=2}^{\infty} (C_{n1} \cos n\omega t + C_{n2} \sin n\omega t) .$$

Assume that the oscillation is close to harmonic: $|C| >> |C_{nj}|$. Using the harmonic balance method, we obtain a first-order solution:

$$A_1(t) = C\cos\omega t + C_{31}\cos 3\omega t,$$

$$C = \sqrt{\frac{4(\omega^2 - \omega_{01}^2)}{3Ka_{11}^2}}, \qquad C_{31} = \frac{Ka_{11}^2}{4(9\omega^2 - \omega_{01}^2)}C^3, \tag{7}$$

$$p_{12}(\omega) = 0 \quad \Rightarrow \quad \omega = \omega', \ \omega = \omega''$$

4.1 Oscillations in the instability region

If the first eigenmode is linearly unstable ($M^* < M < M^{**}$), then $\omega'' \le \omega_{01} \le \omega'$, and the only limit cycle with $\omega = \omega'$ exists.

In order to study transitional solutions consider an energy equation: multiply (6) by $\partial A_1 / \partial t$ and transform to

$$\frac{1}{2}\frac{\partial}{\partial t}\left(\left(\frac{\partial A_1}{\partial t}\right)^2 + \omega_{01}^2 A_1^2 + Ka_{11}^2 \frac{A_1^4}{2}\right) = 2p_2(\omega)\left(\frac{\partial A_1}{\partial t}\right)^2 \tag{8}$$

The left side of (8) is a change of the full oscillation energy. If $p_2(\omega) > 0$, then the energy increases, if $p_2(\omega) < 0$, then it decreases, and only in case of $p_2(\omega) = 0$ the oscillation is neutral.

Consider a physical behavior of the plate. Imagine that a small disturbance (the lowest plate eigenmode) is generated. Linear instability leads to a temporal amplitude growth. Due to non-linear term in (6) the amplitude and the frequency are connected with each other, and the frequency starts to grow too. As free non-linear plate oscillation can have arbitrary amplitude, then non-linearity itself cannot stop the amplitude growth. As a result, growth of the amplitude and the frequency takes place while $p_2(\omega) > 0$. When ω reaches ω', $p_2(\omega)$ becomes equal to zero, and amplification changes into neutral oscillation.

It is easy to see from (8) and figure 1 that the limit cycle with $\omega = \omega'$ is stable. Indeed, increase (decrease) of the amplitude leads to increase (decrease) of the frequency and to a reverse effect from the pressure: decrease (increase) of the energy and following decrease (increase) of the amplitude.

We note that the amplitudes (7) are the same as for free non-linear oscillation of the plate in vacuum. The difference is that in vacuum an oscillation can occur at any frequency (and amplitude), while in a gas flow the frequency is defined by the condition $\omega = \omega'$.

4.2 Leaving the instability region

When $M > M^{**}$, the first eigenmode leaves the instability region: $\omega_{01} \le \omega''$. At the same time the limit cycle oscillation continuously goes on because this cycle is stable. Two stable solutions (rest and the limit cycle oscillation) are separated by the second limit cycle with $\omega = \omega''$, this cycle arises if $\omega_{01} \le \omega''$. It is easy to see from (6) that the second limit cycle is unstable: for arbitrary small decrease of the amplitude the oscillation damps to zero, for arbitrary small increase of the amplitude the oscillation builds up to the first limit cycle with $\omega = \omega'$.

Thus we obtain a qualitative structure of the plate attractors shown in the figure 2, a.

5 EXEMPLARY CALCULATIONS

All plates considered are simply supported, have rigid edges and have no static tension ($M_w = 0$). As for $M = M^*$ $\omega_{01} = \omega'$, then from (5) $\omega_{01} = (M^* - 1)(\pi / L)$. Using this equality and (5), we can rewrite (7) as follows:

$$C = \frac{2\pi\sqrt{M + M^* - 2}}{\sqrt{3Ka_{11}^2}L}\sqrt{M - M^*}, \qquad C_{31} = \frac{2\pi(M + M^* - 2)^{3/2}}{\sqrt{27Ka_{11}^2}L(9(M-1)^2 - (M^* - 1)^2)}(M - M^*)^{3/2} \tag{9}$$

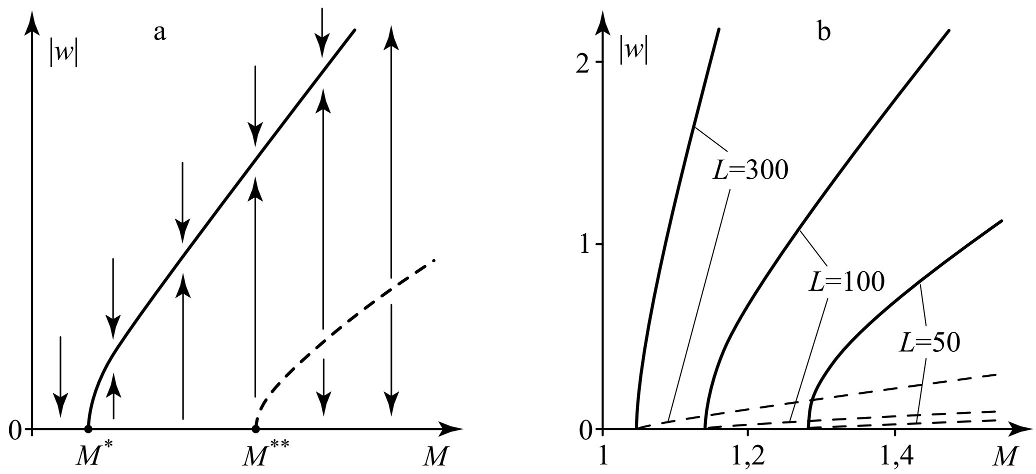

Figure 2. Transitional behavior of a plate in a gas flow (a), amplitudes of the plate deflection divided to its thickness (b). Figure a: solid line is the stable limit cycle, dashed line is the unstable one. Figure b: solid lines are $C(M)$, dashed lines are $C_{31}(M)$; only stable limit cycles are shown.

Due to normalization (3) $W_1(x) = \sqrt{2/L}\cos(\pi x/L)$, therefore in order to obtain "physical" amplitudes (divided to the plate thickness) we should multiply (9) by $\sqrt{2/L}$. Calculations are performed for steel plates in an air with normal conditions ($D = 19.8$, $K = 237$, $\mu = 1.2 \cdot 10^{-4}$). Amplitudes (9) multiplied by $\sqrt{2/L}$ for L=50, 100, 300 are shown in the figure 2, b. We can see that $|C| >> |C_{31}|$, and the assumption made above is correct.

6 CONCLUSIONS

Non-linear analysis of a plate motion in a supersonic gas flow is performed in case of high-frequency flutter. The pressure acting on the plate is calculated with use of results of a linear stability analysis [1, 2] based on the exact theory of potential flow.

The dependence of limit cycle amplitude on the problem parameters is obtained analytically. It is shown that oscillation in a gas flow is the same as in vacuum, but in vacuum the frequency (and consequently the amplitude) is arbitrary, while in a gas the frequency of limit cycle oscillation is defined by the condition $\omega = \omega'(M)$. Exemplary calculations are performed.

ACKNOWLEDGMENTS

The work is supported by Russian Foundation for Basic Research (05-01-00219) and by the grant of the President of Russian Federation for the Support of Leading Scientific Schools (NSh-4710.2006.1).

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