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Numerical Investigation of Supersonic Plate Flutter Using the Exact Aerodynamic Theory

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Abstract—In classical investigations of panel flutter it is usually assumed that the gas pressure acting on the plate can be calculated within the framework of the piston theory, an approximation exact for high Mach numbers. The loss of stability revealed in these investigations is of the "coupled" type, involving the interaction of two oscillation modes. Recently, the use of asymptotic methods revealed another single-mode type of stability loss, which cannot be obtained within the framework of the piston theory. In the present study this type of stability loss is investigated numerically using the Bubnov–Galerkin method.

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Keywords: panel flutter, plate flutter, high-frequency flutter, single-mode flutter, flutter with a single degree of freedom.

Panel flutter, the oscillatory loss of stability of aircraft panels in the form of flat plates or shallow shells, has been investigated in numerous studies (see, for example, [1–6]). In the vast majority of these studies the "piston theory", an approximation at large Mach numbers of the exact expression for the pressure acting on an oscillating plate. Complications such as nonlinearity, inhomogeneity, or complex plate geometry in plan are usually introduced only into the "elastic" part of the problem.

A comparison of the results of piston theory calculations with experiments reveals very good agreement for Mach numbers exceeding 1.7 [1]. With decrease in the Mach number the difference between theory and experiment increases significantly. In order to explain this effect, various suggestions have been made [3, 4]. In particular, it was conjectured that the piston theory loses accuracy as a result of the development of "flutter with a single degree of freedom" [7] or single-mode flutter when a single oscillation mode loses stability without interaction between modes (in contrast to what the piston theory predicts [8, 9]). However, in spite of isolated publications [10–12], panel flutter at low supersonic Mach numbers appears not to have been systematically studied.

Recently, using asymptotic methods, single-mode flutter was detected analytically (in these studies it is called "high-frequency" flutter) [13–15]. It is of interest to investigate the possibility of development of this flutter type numerically, using the exact aerodynamic theory, at low supersonic Mach numbers when the piston theory is not applicable.

1. FORMULATION OF THE PROBLEM

We will consider a flat elastic plate with a plane-parallel supersonic inviscid perfect gas flow on one side. On the other side the plate is subjected to a constant balancing pressure (Fig. 1). The plate is embedded in an absolutely rigid plane that separates the gas flow from the constant-pressure region.

We will consider the problem in the two-dimensional formulation and will assume that the plate has the form of a strip in plan and the incident flow is perpendicular to the edges. We will also assume that the plate oscillates with a deflection $w(x,\ t) = W(x)e^{-i\omega t}$, where ω is the complex frequency (the deflection is divided by the thickness). The dimensionless equation of the plate motion in the gas flow can then be

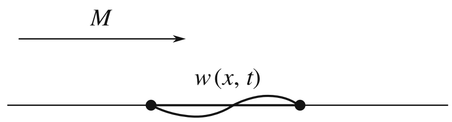

Fig. 1. Diagram of the system considered.

written as follows:

$$D\frac{\partial^4 W}{\partial x^4} - \omega^2 W + p\{W, \omega\} = 0. \tag{1.1}$$

The pressure $p\{W, \omega\}$ found from the linearized potential gas flow theory has the form [1, §4.7]:

$$p\{W, \omega\} = \frac{\mu}{\sqrt{\mathrm{M}^2 - 1}}\left(-i\omega + \mathrm{M}\frac{\partial}{\partial x}\right) \times \int_0^x \left(-i\omega W(\xi) + \mathrm{M}\frac{\partial W(\xi)}{\partial \xi}\right) \exp\left(\frac{i\mathrm{M}\omega(x-\xi)}{\mathrm{M}^2 - 1}\right) J_0\left(\frac{-\omega(x-\xi)}{\mathrm{M}^2-1}\right) d\xi$$

or, after transformations,

$$p\{W, \omega\} = \frac{\mu\mathrm{M}}{\sqrt{\mathrm{M}^2 - 1}}\left(-i\omega W(x) + \mathrm{M}\frac{\partial W(x)}{\partial x}\right) + \frac{\mu\omega}{(\mathrm{M}^2-1)^{2/3}}\int_0^x \left(-i\omega W(\xi) + \mathrm{M}\frac{\partial W(\xi)}{\partial \xi}\right) \times \exp\left(\frac{i\mathrm{M}\omega(x-\xi)}{\mathrm{M}^2-1}\right)\left(iJ_0\left(\frac{-\omega(x-\xi)}{\mathrm{M}^2-1}\right) + \mathrm{M}J_1\left(\frac{-\omega(x-\xi)}{\mathrm{M}^2-1}\right)\right) d\xi. \tag{1.2}$$

The dimensionless parameters can be expressed in terms of the dimensional ones by means of the formulas

$$D = \frac{E}{12(1-\nu^2)a^2\rho_m}, \qquad L = \frac{L_w}{h}, \qquad \mathrm{M} = \frac{u}{a}, \qquad \mu = \frac{\rho}{\rho_m}.$$

Here, E, ν, and ρ_m are Young's modulus, Poisson's ratio and the density of the plate material, L_w and h are the plate width and thickness, and u, ρ, and a are the gas density, velocity and speed of sound. The parameters D and L are the dimensionless plate stiffness and width, and M and μ are the Mach number and dimensionless density of the gas.

The plate occupies the region $0 \le x \le L$ and on its edges hinged-support conditions are assigned:

$$W = \frac{\partial^2 W}{\partial x^2} = 0, \qquad x = 0, \quad x = L. \tag{1.3}$$

Problem (1.1)–(1.3) is an eigenvalue problem for the complex frequency ω. The system is unstable if and only if one of the eigenfrequencies ω_n lies in the upper half-plane of the complex plane $\mathrm{Im}\,\omega_n > 0$.

In what follows, for comparison with the exact expression for the pressure (1.2), we will also perform a calculation in accordance with piston theory in the form of the relation

$$p\{W, \omega\} = \frac{\mu\mathrm{M}}{\sqrt{\mathrm{M}^2-1}}\left(-i\omega W(x) + \mathrm{M}\frac{\partial W(x)}{\partial x}\right) \tag{1.4}$$

which can obviously be obtained from the exact expression (1.2) by discarding the integral term.

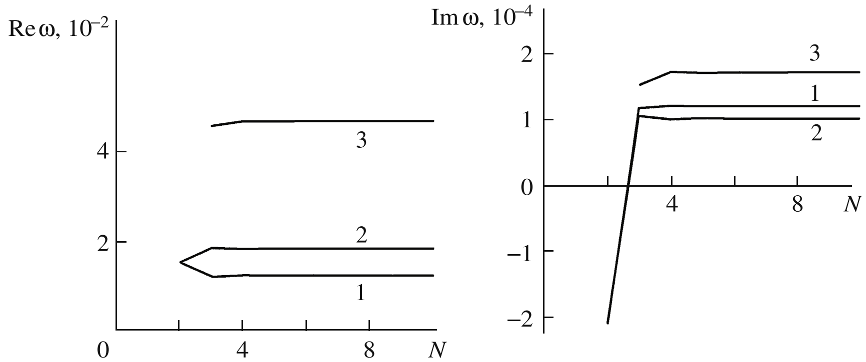

Fig. 2. Convergence of iterations in ω with increase in the number of basis functions, $n = 1$, 2, and 3 are the mode numbers; $D = 23.9$, $\mathrm{M} = 1.2$, $\mu = 1.2 \times 10^{-4}$, $L = 300$.

2. NUMERICAL METHOD

We will solve the eigenvalue problem by the Bubnov–Galerkin method. Taking the plate oscillation shapes in a vacuum as basis functions we will represent the approximate solution in the form:

$$W(x) = \sum_{n=1}^{N} C_n W_n(x), \qquad W_n(x) = \sin\left(\frac{n\pi x}{L}\right).$$

Here, C_n are unknown constant coefficients. Substituting this expression in (1.1), multiplying successively by $W_m(x)$ $(m = 1 \dots N)$ and integrating from 0 to L, we obtain a homogeneous system of linear algebraic equations for C_n with the matrix

$$\mathbf{A}(\omega) = \mathbf{K} + \mathbf{P}(\omega) - \frac{L\omega^2}{2}\mathbf{I}.$$

Here, $\mathbf{K}$ is the diagonal stiffness matrix with the coefficients $k_{jj} = D(j\pi/L)^4(L/2)$ and $k_{jn} = 0$ for $j \neq n$ and $\mathbf{P}$ is the matrix of the aerodynamic forces with the coefficients

$$p_{jn}(\omega) = \int_0^L P\{W_n,\ \omega\} W_j\, dx, \tag{2.1}$$

and $\mathbf{I}$ is a unit matrix. The equation for the eigenvalues has the form:

$$\det \mathbf{A}(\omega) = \det\left(\mathbf{K} + \mathbf{P}(\omega) - \frac{L\omega^2}{2}\mathbf{I}\right) = 0. \tag{2.2}$$

The matrix $\mathbf{P}(\omega)$ is complex and asymmetric, with all the coefficients different from zero. Hence, the eigenvalue problem is not self-conjugate and the eigenfrequencies, solutions of (2.2), are complex.

We will solve Eq. (2.2) numerically using a simple iteration method. Let us calculate the n-eigenfrequency ω_n. As an initial value we will take the n-th plate oscillation eigenfrequency in a vacuum $\omega_{0n} = \sqrt{D}(n\pi/L)^2$. Now let us consider the p-approximation ω_n^p. We construct the matrix $\mathbf{A}_{p+1}(\omega_n^p,\ \omega_n^{p+1})$ so that ω_n^{p+1} enters into it in a simple way. We will take all the matrix coefficients a_{jk}, except for a_{nn}, equal to those of the matrix $\mathbf{A}(\omega_n^p)$ and calculate the coefficient a_{nn} from the formula

$$a_{nn} = k_{nn} + p_{nn}(\omega_n^p) - \frac{L(\omega_n^{p+1})^2}{2},$$

where k_{nn} and p_{nn} are the corresponding coefficients of the matrices $\mathbf{K}$ and $\mathbf{P}$. For the $p+1$-approximation of ω_n^{p+1} we have the equation

$$\det \mathbf{A}_{p+1}(\omega_n^p,\ \omega_n^{p+1}) = 0,$$

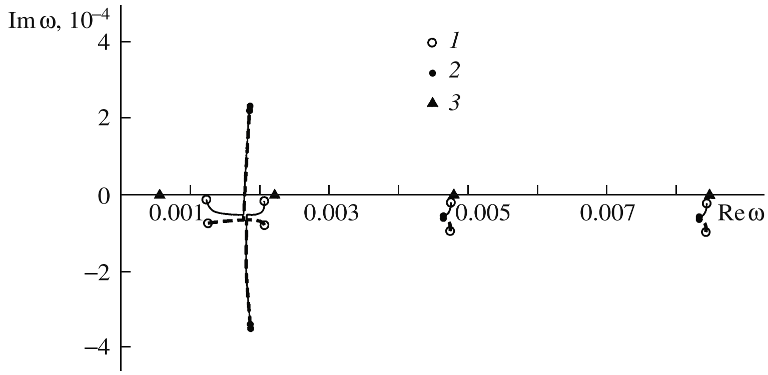

Fig. 3. Trajectories of the first four eigenfrequencies in the complex plane for $D = 23.9$, $\mu = 1.2 \times 10^{-4}$, $L = 300$, and $1.5 \le \mathrm{M} \le 2.7$. Points *1* and *2* are the frequencies for $\mathrm{M} = 1.5$ and 2.7, point *3*—frequencies in a vacuum. The continuous curves are calculated from the exact theory and the broken curves from the piston theory.

which is obviously linear in $(\omega_n^{p+1})^2$. From the two values of ω_n^{p+1} we take the value which lies in the right half-plane of the complex plane: $\mathrm{Re}\,\omega_n^{p+1} > 0$.

The iterations for calculating ω_n are continued until the following condition is satisfied:

$$\frac{\omega_n^p - \omega_n^{p-1}}{\omega_n^p} < \varepsilon_1.$$

After convergence is reached the condition $\det \mathbf{A}(\omega_n^p) < \varepsilon_2$ is checked. In the calculations we took the values $\varepsilon_1 = 10^{-5}$ and $\varepsilon_2 = 10^{-16}$.

In each iteration only the matrix $\mathbf{P}(\omega_n^p)$ is numerically calculated. For calculating each coefficient $p_{jk}(\omega_n^p)$, it is necessary to find two nesting integrals: inner from (1.2) and outer from (2.1). The inner integral is calculated by Simpson's method and the outer one by the trapezoidal rule.

The calculations were performed using five basis functions ($N = 5$). From Fig. 2, which shows the convergence of the numerical method with increase in the number of basis functions, it can be seen that five functions are sufficient to calculate the eigenfrequencies with reasonable accuracy.

3. CALCULATION RESULTS FOR $\mathrm{M} \ge 1.5$

All the calculations were performed for a steel plate in an air flow ($E = 2 \times 10^{11}$ N/m^2, $\nu = 0.3$, $a =$ 300 m/s, $\rho = 1$ kg/m^3, and $\rho_m = 8500$ kg/m^3). The corresponding dimensionless parameters are equal to $D = 23.9$ and $\mu = 1.2 \times 10^{-4}$.

We will first consider the results of calculations based on the exact theory. We fix the plate width $L = 300$ and the Mach number $\mathrm{M} = 1.5$. All the eigenvalues now lie in the lower half-plane (Fig. 3) and the position of the plate is stable. We increase the Mach number leaving all other parameters fixed. The first two eigenfrequencies converge and at $\mathrm{M} = 2.27$ almost coincide. With further increase in M they diverge in directions perpendicular to their trajectories before convergence: the first frequency moves upward and the second downward, almost parallel to the imaginary axis ω. At $\mathrm{M} = 2.29$, that is, almost immediately after merging, the first frequency intersects the real axis and is displaced into the upper half-plane: the situation of the plate becomes unstable. "Coupled" flutter develops due to the interaction of the two lowest eigenfrequencies through an aerodynamic coupling. On the range $1.5 < \mathrm{M} < 2.7$ the third and higher eigenfrequencies vary only slightly and their motion in the complex plane does not lead to instability.

In Fig. 3 the broken curve shows the same trajectories of the eigenfrequencies in the complex plane, but calculated using the piston theory instead of the exact one. Clearly, on the range $\mathrm{M} > 1.5$ the frequencies calculated in accordance with the exact and piston theories are very similar. After the frequencies merge, "coupled"-type stability loss takes place at $\mathrm{M} = 2.30$, that is, the approximate critical Mach number almost

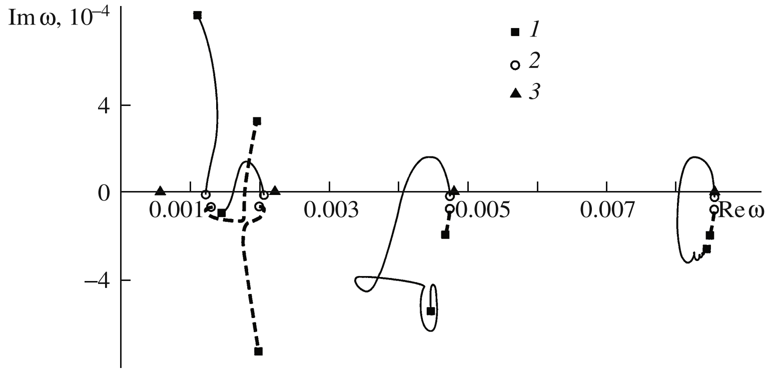

Fig. 4. Trajectories of the first four eigenfrequencies in the complex plane for $D = 23.9$, $\mu = 1.2 \times 10^{-4}$, $L = 300$, and $1.05 \le \mathrm{M} \le 1.5$. Points *1* and *2* are the frequencies for M = 1.05 and 1.5, point *3*—frequencies in a vacuum. The continuous curves are calculated from the exact theory and the broken curves from the piston theory.

coincides with the exact one. With further increase in M the difference between the eigenfrequencies calculated in accordance with the exact and piston theories almost disappears, which again confirms that the piston theory, while being much simpler than the exact one, works well at large M.

4. CALCULATION RESULTS FOR M < 1.5

We will now decrease the Mach number from 1.5 to 1.05, assuming the other parameters to be fixed. As can be seen from Fig. 4, at a certain M_n^{**} the *n*-eigenvalue is displaced into the upper half-plane. The calculated M_n^{**} values are reproduced below:

n	1	2	3	4
M_n^*	< 1.05	1.10	1.10	1.17
M_n^{**}	1.41	1.41	1.44	1.45

The frequencies do not approach one another, that is, there is no interaction of two oscillation modes as in coupled flutter. Thus, this instability is flutter with a single degree of freedom [7] or, in other words, single-mode flutter.

With further decrease in M the oscillation increment of each of the modes reaches a maximum and then begins to decrease. At a certain $\mathrm{M} = \mathrm{M}_n^*$ it becomes negative again and the corresponding mode begins to be damped (the calculated M_n^* are presented above). Thus, on the range $\mathrm{M}_n^* \le \mathrm{M} \le \mathrm{M}_n^{**}$ each of the four modes in question lies in the zone of single-mode instability.

For M < 1.5 the results calculated in accordance with the piston theory and represented in Fig. 4 by the broken curve differ sharply from the results obtained using the exact theory. Firstly, within the framework of the piston theory, single-mode flutter cannot be observed at all. Up to M = 1.10 all the frequencies lie in the lower half-plane. At M = 1.10, a coupled-type flutter develops, with the 1st and 2nd modes involved, but it has no physical meaning. In fact, coupled-type flutter develops when in formula (1.4) the coefficient $\mu \mathrm{M}^2/\sqrt{\mathrm{M}^2 - 1}$ of $\partial W/\partial x$ becomes large. This takes place at both fairly large Mach numbers (like the flutter at M > 2.30 obtained above) and M close to unity.

It was previously known that for M close to unity the piston theory becomes quantitatively incorrect but the above calculations based on the exact theory show that it is also qualitatively incorrect: it “does not see” the single-mode flutter, which actually develops, and yields another coupled-type instability region, which in fact is not there.

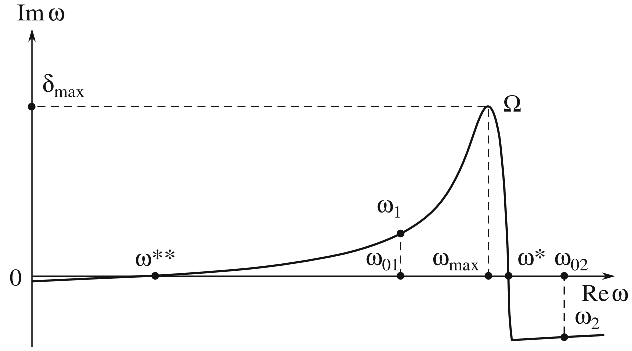

Fig. 5. Typical shape of the curve Ω and construction of the eigenfrequencies from the asymptotic theory.

5. COMPARISON OF THE NUMERICAL AND ASYMPTOTIC RESULTS FOR VARIABLE M

The calculations results revealing two types of stability loss, coupled and single-mode, correlate with the results of [13], where both flutter types (there called "low-" and "high-frequency" flutter) were analytically investigated using the asymptotic global-stability method. Most transparent is the method of calculation of the eigenfrequencies that lie close to the single-mode stability region. Let us consider this method.

In [13] it was shown that for a sufficiently large plate width L the eigenfrequencies of a plate in a gas flow lie near the curve Ω in the complex plane ω and this curve depends on the parameters M, μ, and D but not on L (Fig. 5). Assuming that the gas effect on the absolute value of the eigenfrequencies is slight (since $\mu \ll 1$) and $\operatorname{Im}\omega \ll \operatorname{Re}\omega$, we obtain the following graphic method for calculating the eigenfrequencies. We plot the curve Ω in the complex plane and on the real axis mark off the plate eigenfrequencies in a vacuum. The eigenfrequencies of the plate in the flow can then be found as the projections of the eigenfrequencies in a vacuum on Ω along the straight lines $\operatorname{Re}\omega = \text{const}$ (Fig. 5).

From this there follows the behavior of the frequency trajectories with variation of the problem parameters. With change in the Mach number M the plate frequencies in a vacuum remain unchanged, whereas the curve Ω changes and, moreover, the boundaries of the interval $\omega^{**} < \operatorname{Re}\omega < \omega^{*}$ on which Ω lies in the upper half-plane are displaced:

$$\omega^{*}(\mathrm{M},\, D) \approx \frac{(\mathrm{M}-1)^2}{\sqrt{D}}, \qquad \omega^{**}(\mathrm{M},\, D) = \frac{\mathrm{M}^2 + 1 - \sqrt{4\mathrm{M}^2+1}}{\sqrt{D}}.$$

The above expression for ω^{*} is correct with the order of accuracy $\mu^{2/3}$, ω^{**} is positive for $\mathrm{M} > \sqrt{2}$, and for $\mathrm{M} \le \sqrt{2}$ the curve Ω lies in the upper half-plane over the entire interval $0 < \operatorname{Re}\omega < \omega^{*}$.

Finally, we obtain the following pattern. For sufficiently large M we have $\omega_{0n} < \omega^{**}$ and the n-mode is damped. With decrease in M the points ω^{**} and ω^{*} move leftward, at $\mathrm{M} = \mathrm{M}_n^{**}$ (which can be found from the condition $\omega_{0n} = \omega^{**}$) $\operatorname{Im}\omega$ becomes positive, reaches a maximum, decreases, and at $\mathrm{M} = \mathrm{M}_n^{*}$ (which can be found from the condition $\omega_{0n} = \omega^{*}$) becomes negative again. The same behavior was observed in the numerical calculations for $\mathrm{M} < 1.5$ described above.

The M_n^{**} and M_n^{*} values calculated from the asymptotic theory are presented below:

n	1	2	3	4
M_n^{*}	1.05	1.10	1.15	1.20
M_n^{**}	1.42	1.43	1.44	1.46

At too large M, when $\omega_{0n} \ll \omega^{**}$, the assumption of the small variation of $\operatorname{Re}\omega_n$ with variation of M is incorrect. The link between the different plate modes through the gas flow becomes strong, with increase in M the frequencies approach each other, and coupled-type flutter develops.

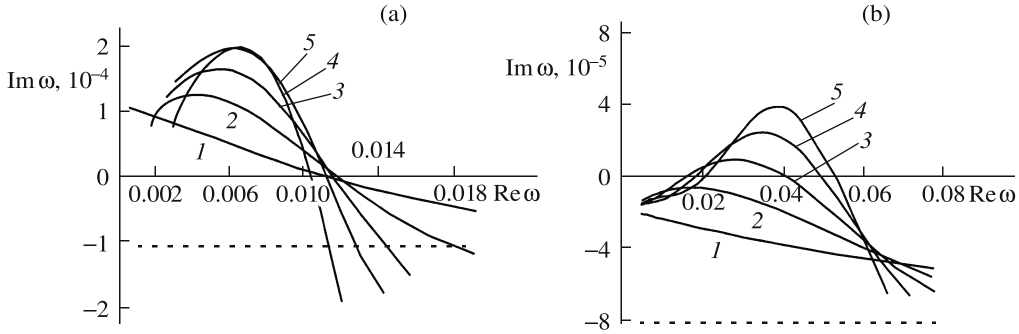

Fig. 6. Trajectories of the first five eigenfrequencies in the complex plane ω for $D = 23.9$, $\mu = 1.2 \times 10^{-4}$, M = 1.2 (a), M = 1.5 (b), and variable L. The continuous curves are calculated from the exact theory and the broken curves from the piston theory (in the latter case all the trajectories coincide). The mode numbers are shown.

6. COMPARISON OF THE NUMERICAL AND ASYMPTOTIC RESULTS FOR VARIABLE L

Especially simple is the behavior of the eigenfrequencies of a plate in a flow that follows from the asymptotic theory for fixed D, M, and μ and variable L. In fact, in this case the Ω position in the complex plane is fixed, while the plate frequencies in a vacuum vary. From this it follows that the trajectories of all the frequencies coincide with each other and Ω.

Figure 6 shows the eigenfrequency trajectories calculated in accordance with the exact aerodynamic theory for fixed parameters $D = 23.9$, $\mu = 1.2 \times 10^{-4}$, M = 1.2 and 1.5 and variable L. For M = 1.2, calculations in accordance with the asymptotic theory yield $\omega^* \approx 0.0082$ and $\omega^{**} = -0.0327$, which means that Ω lies in the upper half-plane on the range $0 < \mathrm{Re}\,\omega < 0.0082$. For M = 1.5 the results are analogous: $\omega^* \approx 0.0511$ and $\omega^{**} = 0.0179$, that is, Ω lies in the upper half-plane on the range $0.0179 < \mathrm{Re}\,\omega < 0.0511$. As can be seen from Fig. 6, the asymptotic and numerical calculations yield similar ω^* and ω^{**} values at M = 1.2 for all modes and at M = 1.5 starting from $n = 4$.

However, despite the relative proximity of the asymptotic and numerical stability limits, the numerically calculated trajectories themselves, that is, the oscillation amplification increments, lie far away from the asymptotic trajectories. For example, the quantity $\delta_{\max}$ (Fig. 5) calculated from the asymptotic theory is equal to 0.00037 for M = 1.2 and 0.00035 for M = 1.5. Clearly, this value is about 1.85 times higher than that obtained numerically from the exact theory for M = 1.2 and 8.75 times higher for M = 1.5. This discrepancy can be explained as follows. The asymptotic solutions [13] were constructed in the form of a linear combination of waves traveling over an imaginary unbounded plate. As a result, all the problem boundary conditions were satisfied except for the impermeability condition on the absolutely rigid plane at $x < 0$. Therefore, the pressures calculated in the neighborhood of the leading edge of the plate in accordance with the asymptotic and exact theories differ sharply, which explains the pronounced difference in $\delta_{\max}$. At the same time, as calculations show, the effect of this difference on the instability itself and its limits is much weaker and at small M > 1 can be neglected.

Summary. The behavior of the eigenfrequencies of a plate in a supersonic gas flow is numerically investigated for various problem parameters. For the gas pressure the exact and piston theories are used.

At low supersonic Mach numbers, within the framework of the exact theory, a single-mode flutter undetected by the piston theory develops. For this flutter type the stability limits are well described by the asymptotic results previously obtained. At the same time, the oscillation amplification increments obtained from the asymptotic theory are overestimated.

At higher Mach numbers a coupled-type flutter develops. Its stability limits calculated from the exact and piston theories almost coincide.

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