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Brief Communication

Effect of damping on flutter of simply supported and clamped panels at low supersonic speeds

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ABSTRACT

Single mode panel flutter is one of two panel flutter types that can occur at low supersonic flow speeds. Over the years it is considered by researchers and engineers as weak and being unable to occur on a real structure due to small growth rate, easily suppressible by the structural damping of the panel. Though recent experiments demonstrated that this opinion is wrong, and single mode flutter can actually occur, it is still unknown what damping level the structure should have to avoid flutter. In this paper we study flutter of damped panels at low supersonic speeds. It is shown that for typical structural damping levels single mode flutter is not always avoidable. Moreover, for some conditions damping level necessary to suppress flutter is too high and cannot be achieved by the structure itself.

1. Introduction

Panel flutter can be caused by two different mechanisms of flow–structure interaction. The first, and most studied mechanism consists in coupling of two structural eigenmodes via the gas flow. In accordance with that, corresponding instability is called coupled-mode flutter. In terms of eigenfrequencies loci, in point of stability-flutter transition two eigenfrequencies coalesce, giving another name to this phenomenon: coalescence flutter. This type of flutter has been studied in numerous (more than 700) papers by using piston theory for modelling of unsteady aerodynamic flow; we mention only principal publications by Movchan (1957), Bolotin (1963), Dowell (1974), Mei et al. (1999) and Algazin and Kiiko (2003). An interesting paper was recently published by Visbal (2012), who studied panel flutter in the presence of an impinging oblique shock wave.

The other flutter mechanism consists in amplification of bending waves by the flow, with no mode coupling and no significant change of the mode shapes. This type of flutter is usually called single mode flutter, or single degree of freedom flutter, and sometimes referred to negative aerodynamic damping of the plate. It is studied much less: the only papers appeared in literature are those by Nelson and Cunnigham (1956), Lock and Farkas (1965), Dowell (1974), Yang (1975), Bendiksen and Davis (1995), Gordnier and Visbal (2001) and Hashimoto et al. (2009). This is mainly because of two reasons. First, single mode type of flutter typically occurs at low supersonic speeds, where piston theory is not valid, and one needs to use much more complicated aerodynamic models (potential flow, Euler, Navier–Stokes equations, etc.) Second, single mode flutter is usually considered as "weak", i.e. unable to occur on real structures because of small growth rate: it is almost always suppressed by the structural damping.

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However, recent experimental investigations (Vedeneev et al., 2010) demonstrated that single mode flutter can actually occur, moreover, several eigenmodes were simultaneously excited. Numerical study (Vedeneev, 2012), where panel damping was not taking into account, showed that single mode flutter can indeed occur in several eigenmodes, such that each of them has its own flutter boundary.

From practical point of view it is important to know influence of structural damping. First, to understand, for which conditions growth rate is small, and single mode flutter indeed will not occur. Second, structural damping is the primary way to suppress single mode flutter, and aircraft designers need to know how much damping the structure must have to avoid flutter. In literature, only Nelson and Cunnigham (1956) and Yang (1975) presented results of flutter calculation of damped panels for conditions of low supersonic speeds. However, they calculated flutter boundaries only for first two eigenmodes and only for Mach numbers 1.3 and 1.41, which is not enough for comprehensive analysis of damping influence. Thus the goal of this paper is investigation of the effect of structural damping at low supersonic speeds, where single mode flutter occurs. We use method and code described in Vedeneev (2012) and present results, first, of inclusion of the structural damping into the plate model, and, second, of flutter study of clamped plates.

2. Formulation of the problem

We study linear stability of elastic plate in a uniform gas flow in 2-D formulation (Fig. 1). Properties of the plate are defined by bending stiffness $\mathcal{D}$, its length $\mathcal{L}$, thickness h, and material density ρ_m; flow parameters are the flow speed u, speed of sound a, and the flow density ρ. For modelling the plate damping we use the simplest model, with damping proportional to the first time-derivative of displacement, and proportionality coefficient equal to twice the damping coefficient G. In-plane plate loads, as well as viscosity of the gas flow are neglected. Corresponding five dimensionless parameters have the form

$$D = \frac{\mathcal{D}}{a^2 \rho_m h^3}, \quad \gamma = G\frac{h}{a}, \quad L = \frac{\mathcal{L}}{h}, \quad M = \frac{u}{a}, \quad \mu = \frac{\rho}{\rho_m}.$$

Gas flows in the upper half-plane; constant pressure equal to undisturbed flow pressure is set in the lower half-plane (Fig. 1). Dimensionless equation of the plate motion has the form

$$D\frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} + 2\gamma\frac{\partial w}{\partial t} + p(x,t) = 0,$$

where w is dimensionless plate deflection (non-dimensionalised by h); p is the gas pressure disturbance.

Consider harmonic motion of the plate, $w(x,t) = W(x)e^{-\mathrm{i}\omega t}$, then the plate equation transforms to

$$D\frac{d^4 W}{dx^4} - (\omega^2 + 2\mathrm{i}\gamma\omega)W + p\{W,\omega\} = 0. \tag{1}$$

Due to neglecting viscosity of the flow, its perturbed state is always potential. Potential flow theory (Miles, 1959) yields the following expression for the pressure disturbance:

$$p\{W,\omega\} = \frac{\mu M}{\sqrt{M^2-1}}\left(-\mathrm{i}\omega W(x) + M\frac{dW(x)}{dx}\right) + \frac{\mu\omega}{(M^2-1)^{3/2}}\int_0^x \left(-\mathrm{i}\omega W(\xi) + M\frac{dW(\xi)}{d\xi}\right) \times \exp\left(\frac{\mathrm{i}M\omega(x-\xi)}{M^2-1}\right)\left(\mathrm{i}J_0\left(\frac{-\omega(x-\xi)}{M^2-1}\right) + MJ_1\left(\frac{-\omega(x-\xi)}{M^2-1}\right)\right)d\xi. \tag{2}$$

We will consider plates simply supported at both edges,

$$W = \frac{d^2W}{dx^2} = 0, \quad x = 0, \quad x = L, \tag{3}$$

or clamped at both edged,

$$W = \frac{dW}{dx} = 0, \quad x = 0, \quad x = L. \tag{4}$$

The problem (1) and (2) with boundary conditions (3) or (4) is the eigenvalue problem for the plate in the gas flow. Positive imaginary part of any eigenfrequency ω_n signifies flutter in the corresponding eigenmode.

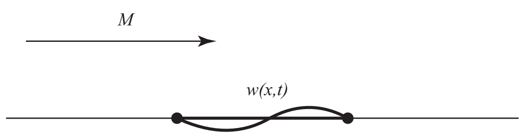

Fig. 1. Plate in supersonic gas flow.

3. Numerical method

Solution method for integro-differential Eq. (1) and (2) has been described and tested in Vedeneev (2012) for the case of $\gamma=0$. The same method is used in this paper for $\gamma\neq 0$, that is why here we will give just a brief overview. We use Bubnov–Galerkin method; basic functions are natural modes of the plate in vacuum. Namely,

$$W(x)=\sum_{n=1}^{N}C_nW_n(x),\quad W_n(x)=\sin\left(\frac{\chi_n x}{L}\right),\quad \chi_n=\pi n,$$

in case of simply supported plate, and

$$W(x)=\sum_{n=1}^{N}C_nW_n(x),\quad W_n=\frac{1}{\sqrt{2}}\left(\cos\frac{\chi_n x}{L}-\cosh\frac{\chi_n x}{L}-\frac{\cos\chi_n-\cosh\chi_n}{\sin\chi_n-\sinh\chi_n}\left(\sin\frac{\chi_n x}{L}-\sinh\frac{\chi_n x}{L}\right)\right),$$
$$\chi_1\approx 4.73,\quad \chi_2\approx 7.859,\quad \chi_n\approx\frac{\pi(2n+1)}{2},n>2,$$

in case of clamped plate. Here C_n are unknown constants. Substitution of this expression into (1), multiplication by $W_m(x)$, $m=1\ldots N$, and integration from 0 to L yields a homogeneous system of algebraic equations with unknowns C_n. Basic functions W_n are normalised such that

$$\int_0^L W_n(x)W_m(x)\,dx=\frac{L}{2}\delta_m^n,$$

therefore the matrix of this system is

$$\mathbf{A}(\omega)=\mathbf{K}+\mathbf{P}(\omega)-\frac{L(\omega^2+2i\gamma\omega)}{2}\mathbf{I}.$$

Here $\mathbf{K}$ is diagonal stiffness matrix with coefficients

$$k_{jj}=D\left(\frac{\chi_j}{L}\right)^4\frac{L}{2},\quad k_{jn}=0,\quad j\neq n,$$

$\mathbf{P}$ is aerodynamic force matrix with coefficients

$$p_{jn}(\omega)=\int_0^L P\{W_n,\omega\}\cdot W_j\,dx, \tag{5}$$

and $\mathbf{I}$ is the unit matrix. Frequency equation, therefore, takes the form

$$\det\mathbf{A}(\omega)=\det\left(\mathbf{K}+\mathbf{P}(\omega)-\frac{L(\omega^2+2i\gamma\omega)}{2}\mathbf{I}\right)=0. \tag{6}$$

An iterative procedure used for solving this equation, and convergence study were described in Vedeneev (2012). The only feature necessary to be mentioned here is that iterations for each n start from the plate eigenfrequency in vacuum $\omega_n^0=\sqrt{D\chi_n^4/L^4}$.

4. Results

Calculations have been conducted for steel plates in air flow at 3000 m above sea level. This corresponds to the following dimensionless parameters: $D=23.9$, $\mu=12\times 10^{-5}$. Three other parameters were varied. Five damping coefficients γ were analysed: 0, 4×10^{-5}, 8×10^{-5}, 12×10^{-5}, 16×10^{-5} (reference values will be given below in Section 4.3). For each of these values stability boundaries (i.e., level lines $\mathrm{Im}\,\omega=0$) for first six modes were calculated and plotted on "Mach number – plate length" plane.

4.1. Simply supported plates

Results are presented in Fig. 2. First of all, it is seen that damping plays much more important role for single mode than for coupled mode flutter. This is in agreement with the mechanism of coupled mode instability. Indeed, in the vicinity of coalescence point of eigenfrequencies the speed of their motion in the complex $\omega-$plane within variation of parameters is very large; this speed is reduced by damping, but stays large. Therefore, small damping can only slightly shift boundary of the coupled mode flutter.

On the contrary, influence of damping on single mode flutter boundaries is essential. First, the maximum length $L_{\max}$ that the plate can have to avoid single mode flutter at any M increases. Second, region of Mach numbers, where panel of each certain length flutters, narrows for each mode. Nevertheless, even for sufficiently high damping $\gamma=16\times 10^{-5}$ there is still

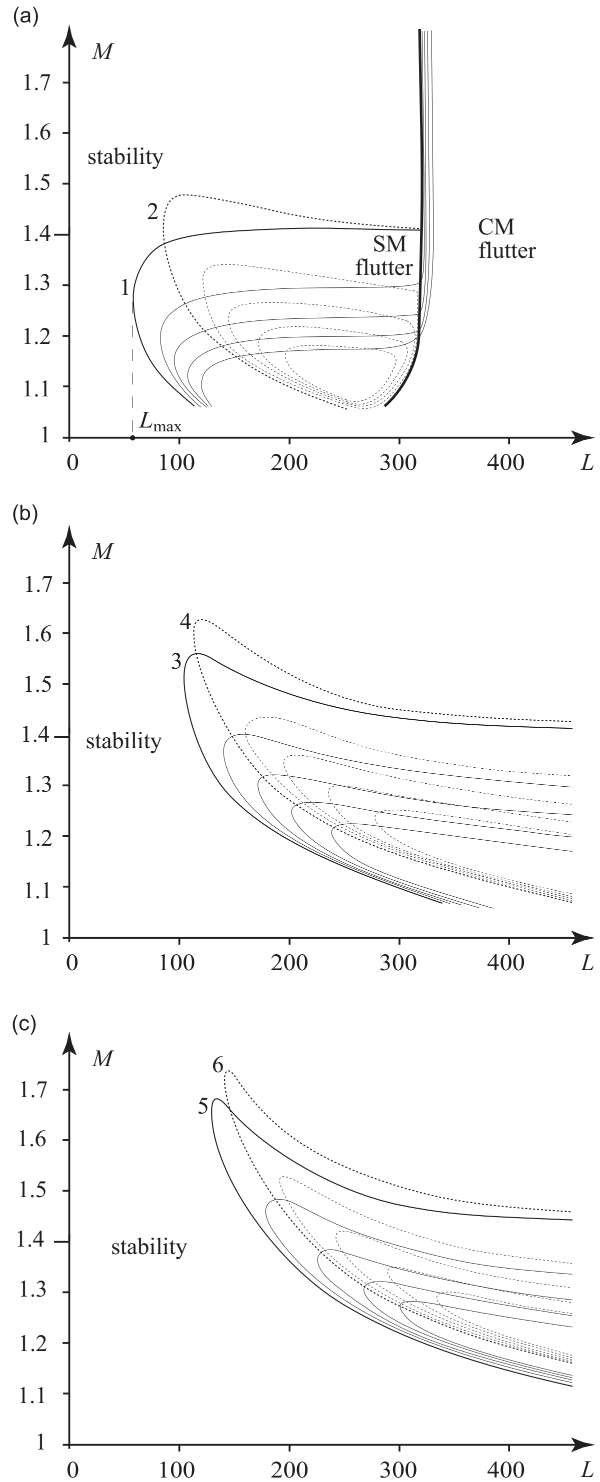

Fig. 2. Stability boundaries of simply supported plate in M–L parameter plane for $D=23.9$, $\mu = 12 \times 10^{-5}$, $\gamma = 0$ (thicker curves), 4×10^{-5}, 8×10^{-5}, 12×10^{-5}, 16×10^{-5} of (a) 1st and 2nd modes, (b) 3rd and 4th modes, (c) 5th and 6th modes. Bold curve in plot (a) delimits single mode (SM) and couple mode (CM) types of flutter.

a region of plate lengths, where flutter is not suppressed. Especially it is seen for the first mode: maximum plate length, for which flutter is avoided for any M, is increased by damping from $L_{max} \approx 57$ at $\gamma = 0$ to $L_{max} \approx 118$ at $\gamma = 16 \times 10^{-5}$, which is still too small for many applications. For instance, panel of $L = 150$ length and $\gamma = 16 \times 10^{-5}$ flutters in $1 < M < 1.15$ range.

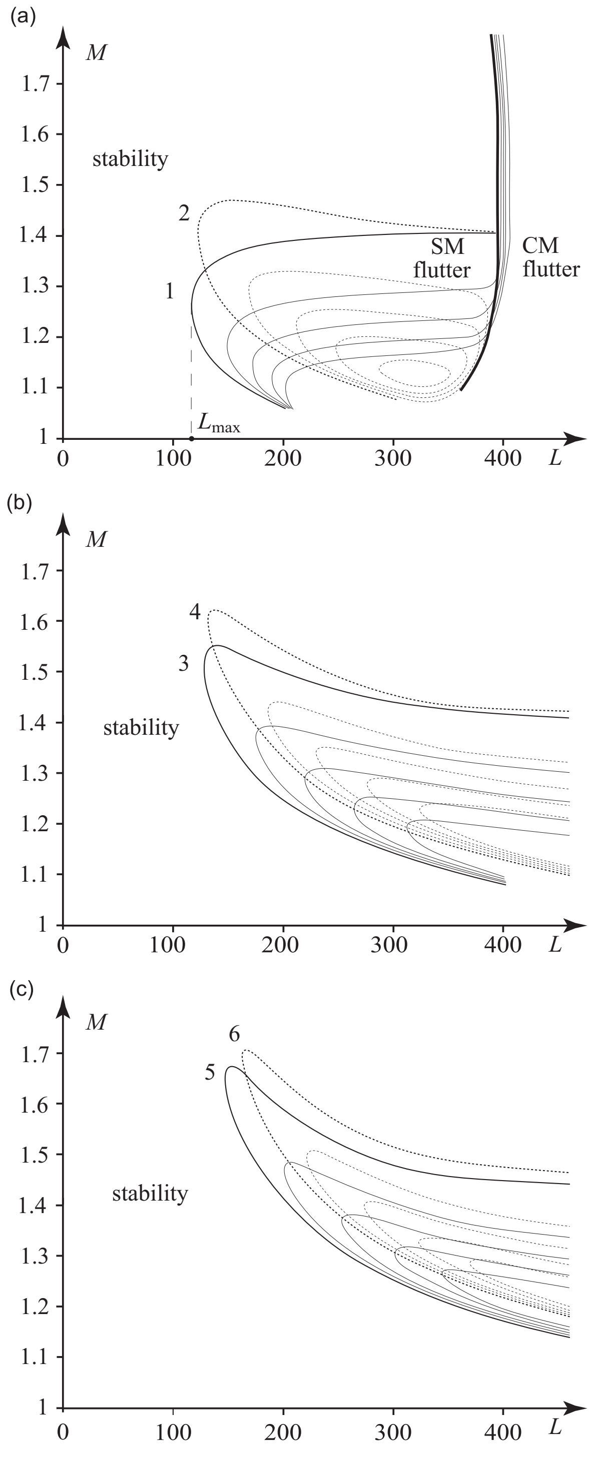

Fig. 3. Stability boundaries of clamped plate in M–L parameter plane for D=23.9, $\mu = 12 \times 10^{-5}$, $\gamma = 0$ (thicker curves), 4×10^{-5}, 8×10^{-5}, 12×10^{-5}, 16×10^{-5} of (a) 1st and 2nd modes, (b) 3rd and 4th modes, (c) 5th and 6th modes. Bold curve in plot (a) delimits single mode (SM) and couple mode (CM) types of flutter.

4.2. Clamped plates

Calculation results for clamped plate are shown in Fig. 3. Both single mode and coupled mode flutter boundaries are shifted to higher L comparing to the case of simply supported plate. General behaviour of the boundaries with change of γ is similar. L_{max} is increased by damping more considerably: from $L_{max}\approx 115$ at $\gamma=0$ to $L\approx 201$ at $\gamma=16\times 10^{-5}$. Therefore, the effect of damping is higher for clamped than for simply supported plates.

4.3. Discussion

Let us now consider some examples in order to understand how much the damping is in typical structures. For clamped metal panel, 0.3×0.001 m size, the first natural frequency $\Omega_1\sim 60$ Hz. Let the panel Q-factor be 10, and air speed of sound be 328 m/s (value at 3000 m above sea level), then

$$G=\frac{\pi\Omega_1}{Q}\sim 19\ \text{Hz} \quad\Rightarrow\quad \gamma=G\frac{h}{a}\approx 5.8\times 10^{-5}.$$

For second and third natural frequencies (167 and 327 Hz, respectively) the dimensionless damping coefficient $\gamma=16\times 10^{-5}$ and 31×10^{-5}.

For higher Q-factors damping coefficients are obviously lower. For example, at $Q=50$ we obtain $\gamma=1.2\times 10^{-5}$, 3.2×10^{-5}, and 6.3×10^{-5} for the first three natural modes. For panels of, say, $L=200$ length this is not enough to suppress flutter in the first and second modes, but sufficient for suppression of flutter in higher modes.

In the calculations above we considered plates made of steel, which gives $\mu=12\times 10^{-5}$. It is important to keep in mind that single mode flutter growth rate is proportional to μ, except a vicinity of lower branches in Figs. 2 and 3, where it is proportional to $\mu^{2/3}$ (Vedeneev, 2006). For metals typical for aeronautical applications this means that more damping is necessary to suppress flutter. For the same flight conditions titanium and aluminium panels yield approximately twice and quadruple μ, respectively, due to change of the metal density. Consequently, twice and quadruple damping coefficients are necessary to suppress flutter (that is, two and four times less Q-factors). Even more damping is necessary for higher flow density at low flight altitude. Such a high damping can be difficult to obtain without special panel dampers or more complicated ways of flutter suppression, such as considered by Zhou et al. (1995). We therefore conclude that common opinion that single mode panel flutter is "weak" and can be neglected by designers and mechanical engineers is erroneous. For conditions of low supersonic speeds it must be taken into account for the design of flutter-free aircraft.

5. Character of apparent plate oscillations

Let us now discuss the spatial oscillations of the plate. It is known that when the flow is transonic, nonlinear limit cycle oscillations have a significant travelling-wave component (Bendiksen and Davis, 1995). When Mach number is large, limit cycle oscillations occur in the form of standing waves. In order to analyse transition from travelling to standing wave when increasing M from linear theory point of view, for each calculated eigenfrequency ω_m we calculated the corresponding eigenmode, namely, amplitudes of basic functions C_n, $n=1\ldots N$ (see Section 3). As the eigenmodes can be arbitrarily scaled, we normalised amplitudes C_n such that $C_m=1$, where m is the number of the eigenmode under consideration. After rewriting in polar form, $C_n=|C_n|e^{i\alpha_n}$, $n\neq m$, it is seen that deviation of α_n or $\beta_n=\alpha_n-\pi$ from zero can be used as a measure of the travelling-wave component.

As an example, consider simply supported plate of length $L=150$ and $\gamma=8\times 10^{-5}$. Plotted in Fig. 4(a) is the amplitude and phase shift of the second-mode component of the first eigenmode ($m=1$). Amplitudes of higher-mode components ($n>2$) do not exceed 0.003 and hence are neglected. It is seen that the lower M is, the more amplitude and phase shift the second-mode component has, which gives travelling-wave behaviour of the eigenmode. For the second eigenmode ($m=2$) amplitudes and phase shifts of the first and third mode components are shown in Fig. 4(b). Though their amplitudes slightly decrease when M decreases, considerable growth of phase shifts β_1 and β_3 is clearly seen. Amplitudes of higher mode components ($n>3$) do not exceed 0.004 and can be neglected.

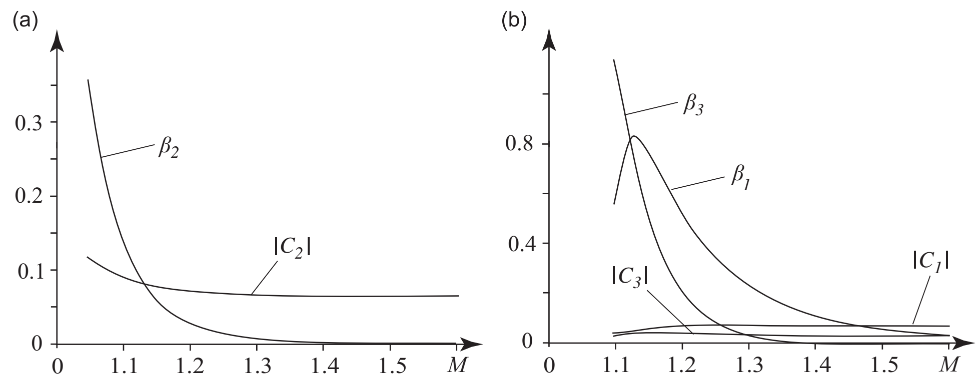

Fig. 4. Amplitudes and phase shifts of eigenmode components for $L=150$, $\gamma=8\times 10^{-5}$: (a) second-mode component of the first eigenmode, (b) first and third-mode components of the second eigenmode.

Similar amplification of travelling-wave components of eigenmodes as $M \to 1$ is detected for other plate lengths and damping coefficients. We therefore conclude that travelling-wave behaviour of limit cycle oscillations at transonic speeds is not only due to nonlinear effects, but is also seen in linear approximation.

6. Conclusions

Calculations of panel flutter boundaries at low supersonic speeds have been conducted with panel damping taken into account. Two boundary conditions are analysed: panels simply supported at both edges and clamped at both edges. By considering typical examples it is shown that conviction of some researchers and engineers to ignore single mode flutter as being “weak” or not able to occur on aircraft is incorrect. Moreover, for sufficiently long panels or light materials special dampers or other mechanisms of damping are necessary to suppress single mode flutter at low supersonic speeds.

Transition from travelling to standing wave behaviour of the plate when Mach number increases has been analysed. It is shown that even in linear approximation significant travelling-wave component appears at transonic flow speeds.

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