7^{th} International Berlin Workshop on
Transport Phenomena with Moving Boundaries and More
30^{th}-31^{st} October 2014, Berlin, Germany

WAVES IN A VISCOELASTIC LAYER INTERACTING WITH A FLUID FLOW

VASILY VEDENEEV

- 1 Leninskie Gory, Department of Hydromechanics, Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Moscow, Russia

Abstract. Compliant viscoelastic coating is an efficient passive way of turbulent friction reduction in boundary layers over bodies moving in a fluid. Wave properties of such coatings play an important role in the interaction with the fluid. For measuring material properties and producing compliant coatings, special testing machines are used. However, wave properties measured in air can differ from those in a moving fluid, and this difference should be included in the process of coating testing. In this paper we consider a layer of viscoelastic material, one side of which is fixed and the other is contacting with a layer of moving fluid. We derive a dispersion relation for waves travelling both in viscoelastic layer and fluid. Influence of the fluid layer thickness and fluid velocity on the wave phase speeds is analysed. Based on the results obtained, correction of method for prediction of compliant coating properties is given.

Keywords. Compliant coatings, boundary layer control, drag force, turbulent friction, fluid-structure interaction.

INTRODUCTION

Reduction of drag force is a challenging hydrodynamic problem for bodies moving in a fluid. Over the years, several efficient ways of boundary layer control have been proposed: suction or blowing of the fluid, surface porosity, riblets, oscillation of the surface, cavitation, etc. Compliance of the surface is also a perspective method of passive flow control. Starting from the work [1], a series of papers is devoted to the investigation of laminar boundary layer stability over compliant surfaces [2–10]. It has been shown that elastic and viscous properties of the surface can significantly change the shape of the neutral stability curve and can change instability character from convective to absolute. Also, in addition to inviscid inflection-point instability and viscous Tollmien-Schlichting instability, a series of new instability types appears due to flexibility of the surface [2, 3].

A series of studies is devoted to drag reduction by compliant coatings in a turbulent flow [11–15]. In contrast to laminar boundary layer, there is no common theoretical approach for modelling the unsteady interaction of turbulent boundary layer with compliant surface. Based on earlier experimental data, the authors proposed a method [12] for choosing optimal properties of the coating that efficiently interacts with the flow at a given velocity. Experiments in air flow [14] proved drag reduction for flows over such coatings. However, the effect of the coating was small due to small flow-to-coating density ratio, and consequently small influence of fluid-structure interaction on the drag reduction. More significant effect is expected in the experiments in water, whose density is of the same order as the coating density. However, in case of water, the method [12] needs corrections. The nature of this method is in measuring wave properties of the coating using a special test facility and calculation of dynamic Young's modulus, Poisson coefficient and viscosity of the coating. In case of coatings intended for water, wave properties will be different when measured in air and in water flow due to influence of the inertia of water. In this paper we give analytical solution for wave properties of a layer of viscoelastic material (coating) in a water layer. We derive the disperion relation and consider examples in order to estimate influence of the water flow on the wave properties of the coating.

FORMULATION OF THE PROBLEM

We study linear waves in a layer of a linear viscoelastic material of the thickness H. On side of the layer is rigidly fixed, while the other is in contact with a layer of inviscid incompressible fluid of the thickness L (fig. 1). The fluid moves with a constant velocity u, which is parallel to the layer. For simplicity, the force of gravity is assumed to be perpendicular to the plane shown in fig. 1. The problem is considered in 2-D formulation so that the gravity is neglected.

Introduce the coordinate system as shown in fig. 1, such that the x axis is directed along the direction of the fluid flow. Since the problem is 2-D, all variables do not depend on z. Let $\{w_x, w_y\}$ be the displacement vector of elastic medium, Φ the perturbation of the potential of

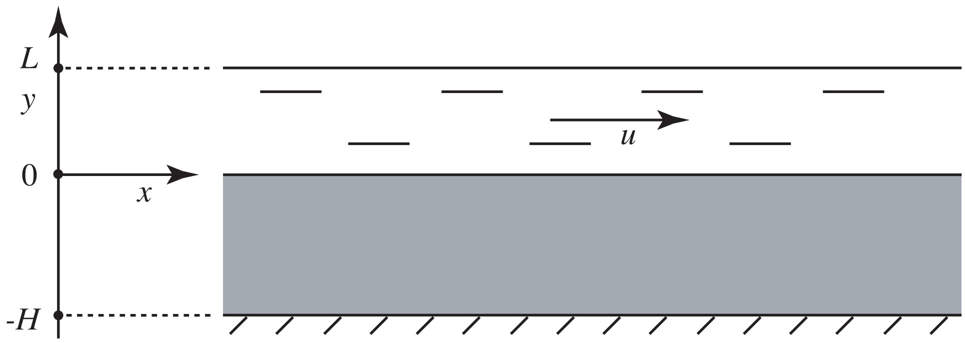

Figure 1: Layer of incompressible fluid over the layer of viscoelastic material.

the fluid. Introduce the functions ϕ and ψ [16], such that

$$w_x = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}, \quad w_y = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}.$$

Since we study oscillatory motions of the system, we use the concept of complex dynamic moduli for modelling viscoelastic properties. Namely, the constitutive equations (Hooke's law) and the governing equations are the same as for linear elastic medium, but Lamé coefficients representing material properties are generally complex. Their real parts represent elastic behaviour, while imaginary parts represent dissipation of energy in oscillatory motion. Expressing strain tensor components through the displacement vector and using Hooke's law, we obtain stress tensor components as functions of ϕ and ψ:

$$\sigma_{22} = \lambda \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + 2\mu \left(\frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x \partial y} \right),$$
$$\sigma_{12} = \mu \left(2\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2} \right),$$

where λ and μ are Lamé coefficients (generally complex, as mentioned above). For convenience, instead of λ and μ we will use speed of longitudinal and transversal wave a_1 and a_2, respectively, as material properties:

$$a_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad a_2 = \sqrt{\frac{\mu}{\rho}} \quad \Rightarrow \quad \lambda = \rho(a_1^2 - 2a_2^2), \quad \mu = \rho a_2^2.$$

We will consider wavy motions of the system:

$$\phi = f(y)e^{i(kx-\omega t)}, \quad \psi = g(y)e^{i(kx-\omega t)}, \quad \Phi = s(y)e^{i(kx-\omega t)}.$$

Then the equations of motion of elastic medium can be written in the form [16]:

$$\begin{aligned} &\frac{d^2 f}{dy^2} - (k^2 - k_1^2) f = 0, \\ &\frac{d^2 g}{dy^2} - (k^2 - k_2^2) g = 0, \end{aligned} \qquad (1)$$

where $k_j = \omega/a_j$. Laplace equation of the fluid motion $\Delta\Phi = 0$ takes the form

$$\frac{d^2 s}{dy^2} - k^2 s = 0. \tag{2}$$

Now let us consider boundary conditions. We will study small perturbations and neglect any second-order terms. At the bottom boundary of the viscoelastic layer we specify zero displacements, $w_x = w_y = 0$:

$$\begin{aligned} &\frac{dg}{dy} + ikf = 0, \quad z = -H, \\ &\frac{df}{dy} - ikg = 0, \quad z = -H. \end{aligned} \tag{3}$$

Along the contact surface between the layer and the fluid, the normal and shear components of stress tensors must be equal:

$$\begin{aligned} &\sigma_{22} = -p = \rho_f \left(u\frac{\partial\Phi}{\partial x} + \frac{\partial\Phi}{\partial t} \right), \quad z = 0, \\ &\sigma_{12} = 0, \quad z = 0. \end{aligned}$$

Using expression for σ_{ij}, we obtain:

$$\begin{aligned} &(a_1^2 - 2a_2^2)\left(\frac{d^2 f}{dy^2} - k^2 f\right) + 2a_2^2\left(\frac{d^2 f}{dy^2} - ik\frac{dg}{dy}\right) = im(ku - \omega)s, \quad z = 0, \\ &\frac{d^2 g}{dy^2} + 2ik\frac{df}{dy} + k^2 g = 0, \quad z = 0, \end{aligned} \tag{4}$$

where $m = \rho_f/\rho$.

Also, along the contact surface the linearised impenetrability condition must be satisfied:

$$\frac{\partial\Phi}{\partial y} = u\frac{\partial w_y}{\partial x} + \frac{\partial w_y}{\partial t}, \quad z = 0,$$

which transforms to

$$\frac{ds}{dy} = i(ku - \omega)\left(\frac{df}{dy} - ikg\right), \quad z = 0. \tag{5}$$

Finally, along the free surface the pressure perturbation must be zero. After linearisation this condition takes the form:

$$p = -\rho_f\left(u\frac{\partial\Phi}{\partial x} + \frac{\partial\Phi}{\partial t}\right) = 0, \quad z = L,$$

which, excluding the case $c = \omega/k \neq u$, is transformed to

$$s = 0, \quad z = L. \tag{6}$$

Thus the system of equations (1), (2) and boundary conditions (3), (4), (5), (6) form a complete system.

Let us emphasise that the system obtained does not take the force of gravity into account. This is valid for the compliant coatings oriented vertically so that the force of gravity is perpendicular to the plane of the wave. Otherwise the system becomes more complex, since the deflections of the free surface and the contact surface become additional unknowns with appropriate additional boundary conditions.

EIGENVALUE PROBLEM

General solution of each equation (1), (2) is expressed as a linear combination of exponents:

$$\begin{aligned} f &= c_1 e^{\lambda_1 y} + c_2 e^{-\lambda_1 y}, \\ g &= c_3 e^{\lambda_2 y} + c_4 e^{-\lambda_2 y}, \\ s &= c_5 e^{\lambda_3 y} + c_6 e^{-\lambda_3 y}, \end{aligned} \tag{7}$$

where $\lambda_1 = \sqrt{k^2 - k_1^2}$, $\lambda_2 = \sqrt{k^2 - k_2^2}$, $\lambda_3 = k$. Substitution into the boundary conditions (3), (4), (5), (6) yields the system of linear homogeneous algebraic equations for c_j with matrix A:

$$\begin{pmatrix} ike^{-\lambda_1 H} & ike^{\lambda_1 H} & \lambda_2 e^{-\lambda_2 H} & -\lambda_2 e^{\lambda_2 H} & 0 & 0 \\ \lambda_1 e^{-\lambda_1 H} & -\lambda_1 e^{\lambda_1 H} & -ike^{-\lambda_2 H} & -ike^{\lambda_2 H} & 0 & 0 \\ 2ik\lambda_1 & -2ik\lambda_1 & \lambda_2^2 + k^2 & \lambda_2^2 + k^2 & 0 & 0 \\ a_1^2\lambda_1^2 - (a_1^2 - 2a_2^2)k^2 & a_1^2\lambda_1^2 - (a_1^2 - 2a_2^2)k^2 & -2ia_2^2 k\lambda_2 & 2ia_2^2 k\lambda_2 & -im(ku-\omega) & -im(ku-\omega) \\ i(ku-\omega)\lambda_1 & -i(ku-\omega)\lambda_1 & k(ku-\omega) & k(ku-\omega) & -\lambda_3 & \lambda_3 \\ 0 & 0 & 0 & 0 & e^{\lambda_3 L} & e^{-\lambda_3 L} \end{pmatrix}$$

Equaling its determinant to zero, we obtain the eigenvalue problem. Express this determinant in the following form:

$$\det A = \underbrace{\begin{vmatrix} ike^{-\lambda_1 H} & ike^{\lambda_1 H} & \lambda_2 e^{-\lambda_2 H} & -\lambda_2 e^{\lambda_2 H} \\ \lambda_1 e^{-\lambda_1 H} & -\lambda_1 e^{\lambda_1 H} & -ike^{-\lambda_2 H} & -ike^{\lambda_2 H} \\ 2ik\lambda_1 & -2ik\lambda_1 & \lambda_2^2+k^2 & \lambda_2^2+k^2 \\ a_1^2\lambda_1^2-(a_1^2-2a_2^2)k^2 & a_1^2\lambda_1^2-(a_1^2-2a_2^2)k^2 & -2ia_2^2k\lambda_2 & 2ia_2^2k\lambda_2 \end{vmatrix}}_{\det A_1} \cdot \begin{vmatrix} -\lambda_3 & \lambda_3 \\ e^{\lambda_3 L} & e^{-\lambda_3 L} \end{vmatrix} +$$

$$+ \underbrace{\begin{vmatrix} ike^{-\lambda_1 H} & ike^{\lambda_1 H} & \lambda_2 e^{-\lambda_2 H} & -\lambda_2 e^{\lambda_2 H} \\ \lambda_1 e^{-\lambda_1 H} & -\lambda_1 e^{\lambda_1 H} & -ike^{-\lambda_2 H} & -ike^{\lambda_2 H} \\ 2ik\lambda_1 & -2ik\lambda_1 & \lambda_2^2+k^2 & \lambda_2^2+k^2 \\ i(ku-\omega)\lambda_1 & -i(ku-\omega)\lambda_1 & k(ku-\omega) & k(ku-\omega) \end{vmatrix}}_{\det A_2} \cdot \begin{vmatrix} -im(ku-\omega) & -im(ku-\omega) \\ e^{\lambda_3 L} & e^{-\lambda_3 L} \end{vmatrix}$$

Calculations yield the following expressions:

$$\det A_1 = -4a_2^2k^6\alpha\beta \left[\left(2-\left(\frac{c}{a_2}\right)^2\right)^2 \left(\cosh(\lambda_1 H)\cosh(\lambda_2 H) - \frac{\sinh(\lambda_1 H)\sinh(\lambda_2 H)}{\alpha\beta}\right) + \right.$$
$$\left. + 4\left(\cosh(\lambda_1 H)\cosh(\lambda_2 H) - \alpha\beta\sinh(\lambda_1 H)\sinh(\lambda_2 H)\right) - 4\left(2-\left(\frac{c}{a_2}\right)^2\right)\right],$$

$$\det A_2 = -2ik^6\alpha\left(\frac{c}{a_2}\right)^2 (u-c)\left[(1-\alpha\beta)\sinh((\lambda_1+\lambda_2)H) + (1+\alpha\beta)\sinh((-\lambda_1+\lambda_2)H)\right],$$

where

$$\alpha = \sqrt{1-\left(\frac{c}{a_1}\right)^2}, \quad \beta = \sqrt{1-\left(\frac{c}{a_2}\right)^2}$$

Substituting the expressions for $\det A_j$ into the formula for $\det A$, calculating two-dimensional determinants, and cancelling, we obtain the dispersion relation:

$$2\left[\left(2-\left(\frac{c}{a_2}\right)^2\right)^2\left(\cosh(\lambda_1 H)\cosh(\lambda_2 H) - \frac{\sinh(\lambda_1 H)\sinh(\lambda_2 H)}{\alpha\beta}\right) + \right.$$
$$\left. + 4\left(\cosh(\lambda_1 H)\cosh(\lambda_2 H) - \alpha\beta\sinh(\lambda_1 H)\sinh(\lambda_2 H)\right) - 4\left(2-\left(\frac{c}{a_2}\right)^2\right)\right] =$$
$$= m\left(\frac{c}{a_2}\right)^2\left(\frac{u-c}{a_2}\right)^2\frac{1}{\beta}\tanh(kL)\left[(1-\alpha\beta)\sinh((\lambda_1+\lambda_2)H) + (1+\alpha\beta)\sinh((-\lambda_1+\lambda_2)H)\right] \tag{8}$$

Note that neglecting the right-hand side of the equation (8) yields the dispersion relation for viscoelastic layer in vacuum [12]. Therefore the right-hand side in (8) represents the influence of the fluid layer. It is zero in the following cases:

- $L = 0$: in fact, this means absence of the fluid.
- $m = 0$: this means absence of the inertia of the fluid so that it does not affect oscillations.
- $c = u$: the wave is steady with respect to the fluid. Due to lack of the force of gravity, it does not produce perturbation of the fluid, so that the pressure at the contact surface is zero.
- $c = 0$: the wave is steady with respect to the viscoelastic layer. In such wave $w_y = 0$ along the top surface of the layer (it follows from (4) for $c = 0$), i.e. the surface oscillates only in transverse direction and does not excite the motion of the fluid.

In other cases the fluid affects the waves.

Consider the effect of the fluid layer thickness L. It is present only in the term $\tanh(kL)$, which tends to 1 as $L \to \infty$. This term differs from 1 by less than 1% for $kL > 2.65$. It was shown [12] that in vacuum the wave length $3H$ yields optimal coating-fluid interaction so that we consider here $k = 2\pi/(3H)$ as an example. For a typical coating thichness $H = 0.01$ m this yields $L > 2.65 \cdot 3H/(2\pi) = 0.013$ m. This means that the thickness of the fluid layer has effect only for very thin layers: for fluid layer of the order of $L = 0.01$ m or higher there is no difference with the case of $L = \infty$.

EXAMPLES

Here we study the phase speed c (namely, the first eigensolution of the equation (8)) for the following parameters:

$$a_1 = 22 \text{ m/s}, a_2 = 15 \text{ m/s}, H = 0.005 \text{ m}, L = 1 \text{ m}, m = 1. \tag{9}$$

Note that the value of $L = 1$ m in fact is equivalent to infinite fluid layer, as argued above. Consider oscillations with wave length $3H$, i.e. $k = 2\pi/(3H)$ (it was shown [12] that in vacuum this condition provides maximum normal displacement along the surface of the viscoelastic layer, which implies maximum interaction of the coating with the flow). Denote $F_1(c)$ the left-hand side of (8)), $F_2(c)$ the right-hand side. In the figures below the plots of $F_1(c) - F_2(c)$ (full dispersion relation) and $F_1(c)$ (dispersion relation with the fluid layer neglected) are shown by blue and green curves, respectively. Also, black curves represent the dispersion relation for Rayleigh wave ($H = \infty$, $m = 0$).

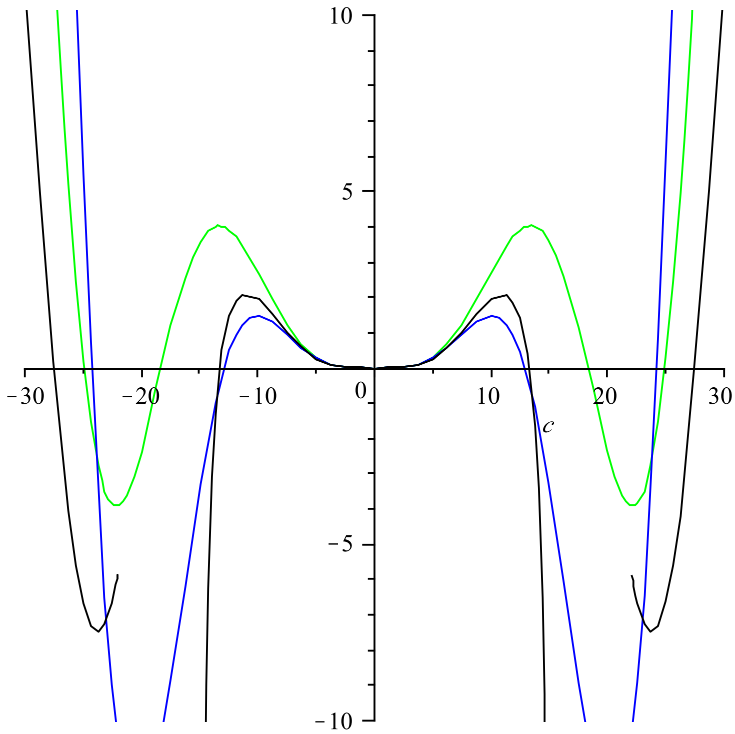

Figure 2: Plot $F_1(c)$ (green), $F_1(c) - F_2(c)$ (blue) for parameters (9), $u = 0$ m/s. Black curve represents the plot of $F_1(c)$ for $H = \infty$, $m = 0$ (Rayleigh waves).

Results for the case of $u = 0$ m/s are shown in fig. 2. It is seen that for the coating in the fluid at rest the wave phase speed is lower than for the coating in vacuum (13 m/s versus 19 m/s). When the flow speed is increased (fig. 3–5), the phase speed of the downstream travelling wave is increased, while the speed of the upstream travelling wave is decreased. For $c \approx u$, the fluid influence is negligible so that the phase speeds for the wave in a fluid and in vacuum almost coinside (fig. 4). For higher flow speed the phase speed decreases, at the same time the second root appears (fig. 5). This additional root corresponded to upstream travelling wave for lower flow speeds, the phase speed was decreasing for flow speed increasing, and finally this wave became downstream-travelling.

Accoring to the criteria for optimal compliant coating [12], maximum interation of the coating with the flow is achieved for $c = 0.7...0.8u$. Since the root c for the wave in fluid is less than for the wave in vacuum, we conclude that optimal flow speed for a given coating is slightly lower in the fluid. That is, the maximum effect of the coating in water is achieved at lower

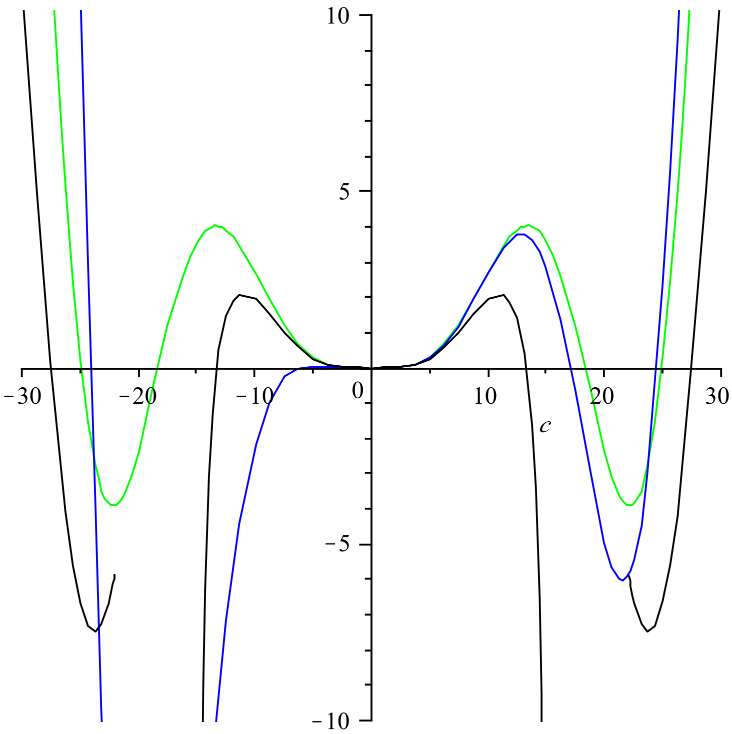

Figure 3: Plot $F_1(c)$ (green), $F_1(c) - F_2(c)$ (blue) for parameters (9), $u = 10$ m/s. Black curve represents the plot of $F_1(c)$ for $H = \infty$, $m = 0$ (Rayleigh waves).

speeds than in air. However, for parameters (9) this difference is actually small. Namely, optimal fluid speed is 22...24.5 m/s for fluid taken into account (case of water) and 23...26 m/s for the fluid neglected (case of air).

When the flow speed is further increased, two downstream-travelling roots approach each other and coalesce. The coalescence occurs at $u \approx 26$ m/s for parameters (9), the double root $c \approx 14$ m/s. For higher u the roots become complex-conjucated so that one of them gives unstable solution which represents travelling-wave flutter of the coating. Seemingly, this situation should be avoided, since high-amplitude vibrations of the coating can yield its damage, more rapid ageing and significantly increased drag force. Note that the optimal flow speed for the case of water is pretty close to flutter speed, which means that the coating should be designed such that the optimal fluid velocity is close enough to the maximum velocity of the body where the coating is installed.

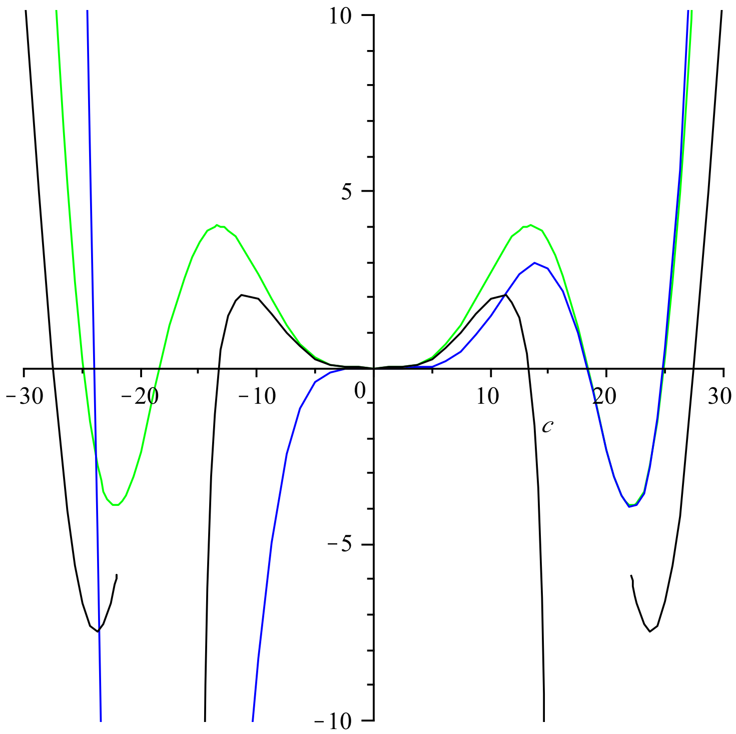

Figure 4: Plot $F_1(c)$ (green), $F_1(c) - F_2(c)$ (blue) for parameters (9), $u = 20$ m/s. Black curve represents the plot of $F_1(c)$ for $H = \infty$, $m = 0$ (Rayleigh waves).

CONCLUSIONS

Influence of a fluid layer on the waves in viscoelastic compliant coatings has been studied. Such coatings can be effectively used for turbulent drag reduction for bodies moving in water. The dispersion relation for sinusoidal waves has been derived. Its analysis based on the examples considered leads to the following conclusions. First, phase speed of the wave in the coating in water is less than the one measured in air. Second, optimal (i.e. providing maximum coating-flow interaction) velocity of water flow is less than the optimal velocity of air flow. Third, for water flow velocities slightly higher than the optimal velocity, the flutter instability of the coating can occur. The latter conclusion means that in order to avoid flutter when operating in water, compliant coating properties should be chosen such that the optimal flow velocity is not much less than the maximum design speed of the body on which the coating is installed.

The work is supported by grants of Russian Foundation for Basic Research (14-01-00049 and 14-01-00052) and grant NSh-3530.2014.1.

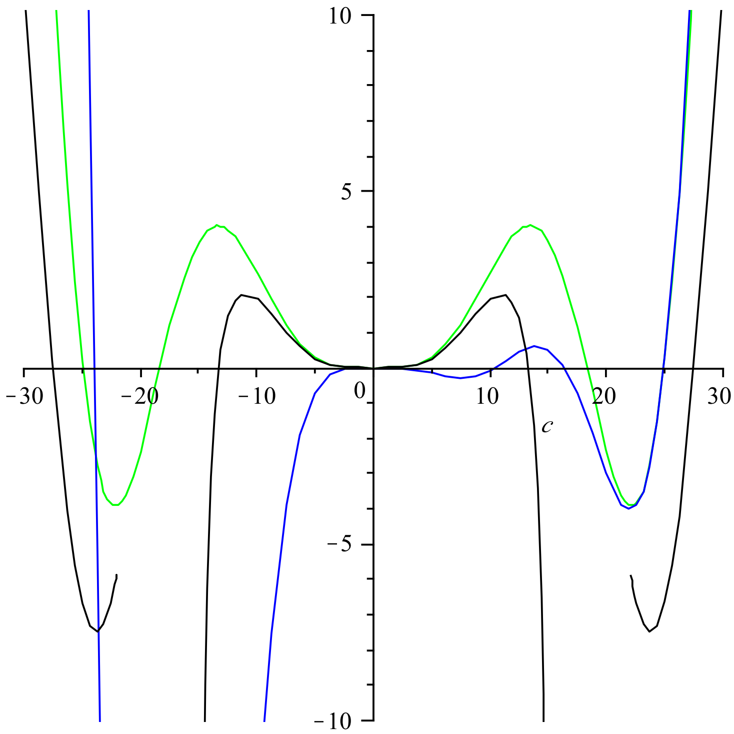

Figure 5: Plot $F_1(c)$ (green), $F_1(c) - F_2(c)$ (blue) for parameters (9), $u = 25$ m/s. Black curve represents the plot of $F_1(c)$ for $H = \infty$, $m = 0$ (Rayleigh waves).

References

[1] Kramer, M. O.: Boundary layer stabilization by distributed damping. J. Amer. Soc. Nav. Engrs., 72 (1960), 25–33.

[2] Carpenter, P. W.; Garrad, A. D.: The hydrodynamic stability of flow over Kramer-type compliant surfaces: Part 1. Tollmien-Schlichting instabilities. J. Fluid Mech., 155 (1985), 465–510.

[3] Carpenter, P. W.; Garrad, A. D.: The hydrodynamic stability of flow over Kramer-type compliant surfaces: Part 2. Flow-induced surface instabilities. J. Fluid Mech., 170 (1986), 199–232.

[4] Savenkov, I.V.: The suppression of the growth of nonlinear wave packets by the elasticity of the surface around which flow occurs. Comput. Math. Math. Phys., 35(1) (1995), 73–79.

[5] Reutov, V.P. ; Rybushkina, G.V.: Hydroelastic instability threshold in a turbulent boundary layer over a compliant coating. Phys. Fluids, 10(2) (1998), 417–425.

[6] Lingwood, R. J. ; Peake, N.: On the causal behaviour of flow over an elastic wall. J. Fluid Mech., 396 (1999), 319–344.

[7] Miles, J.: Stability of inviscid shear flow over a flexible boundary. J. Fluid Mech., 434 (2001), 371–378.

[8] Wiplier, O.; Ehrenstein, U.: On the absolute instability in a boundary-layer flow with compliant coatings. European J. Mech. – B/Fluids, 20(1) (2001), 127–144.

[9] Peake, N.: On the unsteady motion of a long fluid-loaded elastic plate with mean flow. J. Fluid Mech., 507 (2004), 335–366.

[10] Boiko, A. V.; Kulik, V. M.: Stability of flat plate boundary layer over monolithic viscoelastic coatings. Doklady Physics, 57(7) (2012), 285–287.

[11] Bandyopandhy, P.R.; Henoch, C.; Hrubus, J.D.; Semenov, B.N.; Amirov, A.I.; Kulik, V.M.; Malyuga, A.G.; Choi, K.-S.; Escudier. M.P.: Experiments on the effects of ageing on compliant coating drag reduction. Physics of Fluids, 17(8) (2005), 085104.

[12] Kulik, V.M.; Lee, I.; Chun, H.H.: Wave properties of coating for skin friction reduction, Physics of Fluids, 20, 075109 (2008)

[13] Kulik, V.M.; Boiko, A.V.; Bardakhanov, S.P.; Park, H.; Chun, H.H.; Lee, I.: Viscoelastic properties of silicone rubber with admixture of SiO2 nanoparticles. Materials Science and Engineering A, 528(18) (2011), 5729-5732.

[14] Kulik, V.M.; Boiko, A.V.; Chun, B.M.; Lee, I.: Verification of drag-reduction capabilities of stiff compliant coatings in air flow at moderate speeds. International journal of Naval Architecture and Ocean Engineering, 3(4) (2011), 242-253.

[15] Kulik, V.M.: Action of a turbulent flow on a hard compliant coating. Heat and Fluid Flow, 33 (2012), 232-241.

[16] Sedov, L.I.: Mechanics of Continuous Media. V. 2, World Scientific, (1997).