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NUMERICAL ANALYSIS OF SINGLE MODE PANEL FLUTTER IN A VISCOUS GAS FLOW

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ABSTRACT

In this paper single mode panel flutter, which occurs at low supersonic Mach numbers, is studied. Numerical analysis which does not require solution of coupled FSI problem has been conducted. Flutter boundaries obtained are compared with previously known analytical results.

INTRODUCTION

Panel flutter is self-exiting high-amplitude vibrations of elastic plate in a gas flow. Arising at skin panels of flight vehicles, such vibrations can lead to fatique damage of the panels or structures linked to the panels (hydraulic tubes, etc). Panel flutter was discovered in 1940-s and since was thourougly studied in supersonic and hypersonic gas flows. From mathematical point of view, panel flutter problem consists of stability analysis of coupled gas-plate equation (Fig. 1):

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\rho_m h\frac{\partial^2 w}{\partial t^2}+p=0, \quad (1)$$

where D and ρ_m are the plate stiffness and density, $w(x,y,t)$ and h is the plate deflection and thickness, and $p=p(w(x,y,t),x,y,t)$ is gas pressure perturbation, which in turn is a function of w.

At high Mach numbers, M, gas pressure is expressed through a simple approximate formula, known as ”piston theory”, which is valid for $M \gg 1$. By means of this formula, huge amount of studies have been conducted [1, 2]. Type of flutter which occurred at those studies is a ”coupled-type” flutter, which appears due to interaction of two plate eigenmodes. Theory of coupled-type flutter is in a good correlation with experiments at $M > 1.7$.

At low supersonic Mach numbers another flutter type, ”single mode” (or ”single degree of freedom”) flutter, usually occurs. To date it was studied very little and have been mentioned in a few papers [3–5]. However, during last years significant progress in single mode panel flutter study has been achieved. First, asymptotic theory of flutter, leading to a closed-form solution, has been constructed, which brings light to a physical phenomena of single mode flutter through analysis of waves moving along the plate in a gas flow [6, 7]. Second, numerical analysis of 2D problem has been conducted by means of Bubnov-Galerkin method [8]. Flutter boundaries are in a good agreement between numerical and analytical results in all studied range $1.05 < M < 2.7$. Finally, experimental study has been conducted, where single mode flutter was observed [9].

In this paper we study 3D single mode panel flutter in a viscous gas flow using finite volume numerical code. Used is a special method based on single mode flutter properties, which does not require to solve coupled fluid-structure problem. The paper consists of 3 sections. First, we will describe the analysis method, then we will verify the method comparing 2D results with the paper [8]. Finally, we will present results of 3D problem study and compare it with analytical results [7].

NUMERICAL METHOD OF FLUTTER PREDICTION

We study stability of a thin elastic rectangular plate in a plane-parallel supersonic air flow. The plate is mounted into a rigid plane as shown in Fig. 1. Undirsturbed plate is flat, but arbitrary small disturbance lead to oscillation of the plate. These oscillations perturb homogeneous air pressure, and the pressure disturbance in turn affects the plate. Under the action of the pressure disturbance the plate oscillations can be amplified (case of flutter) or damped (case of stability).

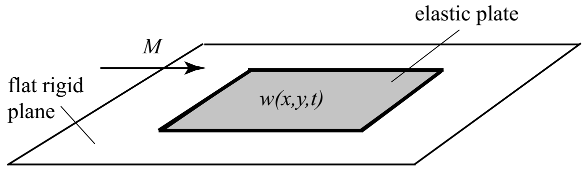

FIGURE 1. GEOMETRY OF THE PROBLEM.

As we are going to study single degree of freedom flutter, we will assume that the flow is supersonic, but Mach number is not very high, so that coupled type flutter is impossible (for typical plates, approximately $M < 1.7$). Therefore only single degree of freedom flutter can occur, which main feature is that the eigenfrequencies and eigenmodes of the plate in the flow are close to eigenfrequencies and eigenmodes of the plate in vacuum.

In order to study stability of the plate in a gas flow, we will use finite-volume code Ansys CFX. Simulation domain is shown in Fig. 2. The plate is assumed to be a part of the domain boundary. In the domain Navier-Stokes equations are solved with noslip condition at the plate and rigid walls. At the inlet the gas pressure velocity and temperature are specified, and no boundary condition specified at the outlet. The plate oscillations are modelled as a motion of the flow boundary. Specified displacement coincides with the natural mode of the plate:

$$w(x,y,t) = W(x,y) \cdot \sin(\omega t),$$

where $W(x,y)$ and ω are plate natural mode and frequency. Thus, the plate motion is modelled as prescribed harmonic motion of the flow boundary.

Oscillation of the plate (in terms of the analysis, motion of the domain boundary) leads to disturbance of the air pressure. Analysis is running until the flow response to harmonic plate motion also becomes harmonic. Then analysis stops, and work done by pressure during last oscillation period is calculated:

$$U = \int_0^T \int_S \vec{p}(x,y,z,t) \cdot \vec{v}(x,y,t) ds dt, \quad (2)$$

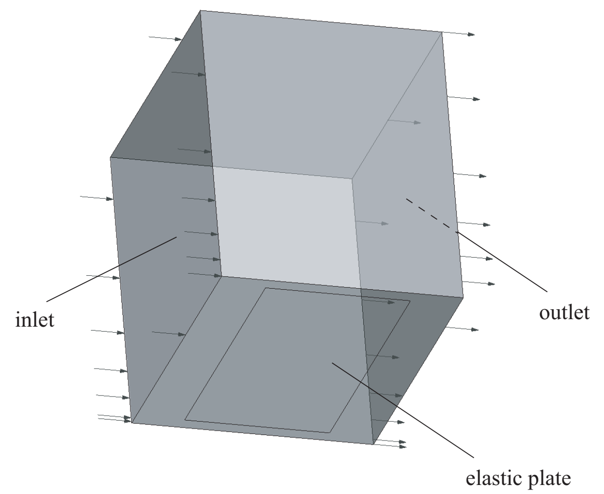

FIGURE 2. SIMULATION DOMAIN.

where $T = 2\pi/\omega$, S is the plate surface, $\vec{p}$ is a pressure acting on the oscillating plate, $\vec{v} = \partial w/\partial t$ is the plate velocity vector.

Sign of U is the plate flutter criterion. Indeed, if $U > 0$, then energy flows from the gas flow to the plate, and the plate oscillations are amplified. In this case plate amplitude increases, and flutter occurs. Otherwise, if $U < 0$, then energy flows from the plate to the flow and dissipates there. The plate damps, and its undisturbed flat state is stable. This criterion should be checked for every possible fluttering mode. If at least at one mode the work (2) is positive, than the plate is unstable.

If we release the plate and let it oscillates itself, amplitude will exponentially increase, if $U > 0$, or decrease, if $U < 0$. Thus motion of the released plate is

$$w(x,y,t) = W(x,y) \cdot \sin(\omega t) e^{\delta t}, \quad (3)$$

where $\delta = \delta(U)$ is the amplitude amplification rate.

We will assume that $|\delta T| \ll 1$ and derive formula for $\delta(U)$. Multiplying Eqn. (1) by $\vec{v} = \partial w/\partial t$ and integrating by x and y along the plate surface, we obtain the energy balance equation:

$$\frac{\partial}{\partial t} E(t) = \int_S \vec{p}(x,y,z,t) \cdot \vec{v}(x,y,t) ds, \quad (4)$$

where

$$E(t) = \frac{1}{2} \int_S \left(D \left(\left(\frac{\partial^2 w}{\partial x^2} \right)^2 + 2 \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 + \right.\right. \quad (5)$$

$$\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right) + \rho_m h \left(\frac{\partial w}{\partial t}\right)^2\right) ds$$

is the total plate energy.

Integrating Eqn. (4) by t, we obtain that at each oscillation period energy of the plate is changed by U:

$$\Delta E(t)_{t=0\ldots T} = U \tag{6}$$

We see again, that if $U \neq 0$, free plate motion is not harmonic. Substitution of Eqn. (3) into Eqn. (5) yields

$$E(t) = \frac{e^{2\delta t}}{2} \int_S \left(D \left(\left(\frac{\partial^2 W}{\partial x^2} \right)^2 + 2 \left(\frac{\partial^2 W}{\partial x \partial y} \right)^2 + \left(\frac{\partial^2 W}{\partial y^2} \right)^2 \right) \sin^2 \omega t + \rho_m h W^2 (\omega^2 \cos^2 \omega t + 2\omega\delta \sin \omega t \cos \omega t + \delta^2 \sin^2 \omega t) \right) ds$$

Expanding $e^{2\delta t} = 1 + 2\delta t$ for $|\delta t| \ll 1$ and omitting terms of order of $(\delta T)^2$ and higher, we obtain:

$$\Delta E(t)_{t=0\ldots T} = \delta \rho_m h T \omega^2 \int_S W^2(x,y) ds.$$

Using Eqn. (6) yields:

$$\delta(U) = \frac{U}{2\pi\omega\rho_m h \int_S W^2(x,y) ds} = \frac{U \cdot T}{4\pi^2 \rho_m h \int_S W^2(x,y) ds}$$

We will analyze plate of $L_x \times L_y$ size simply supported at all edges. Therefore

$$W(x,y) = A \sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right),$$

where $|A| \ll 1$ is the oscillation amplitude, and

$$\int_S W^2(x,y) ds = A^2 \int_0^{L_y} \int_0^{L_x} \sin^2\left(\frac{m\pi x}{L_x}\right) \sin^2\left(\frac{n\pi y}{L_y}\right) dx dy = \frac{A^2 L_x L_y}{4}$$

Finally,

$$\delta(U) = \frac{2U}{\pi\omega\rho_m h A^2 L_x L_y} = \frac{U \cdot T}{\pi^2 \rho_m h A^2 L_x L_y} \tag{7}$$

VERIFICATION OF THE METHOD

In order to verify the method of flutter analysis described above, 2D problem was considered first. A steel plate, 0.3×0.001 m size is studied in air flow. The results have been compared with [8], where the same problem was studied through Bubnov-Galerkin method. In that paper the equation of thin plate bending and potential gas flow were used. It was shown that at parameters considered there are two regions of instability of first 4 modes: single mode flutter at $1 < M < 1.45$ and coupled-type flutter at $M > 2.29$.

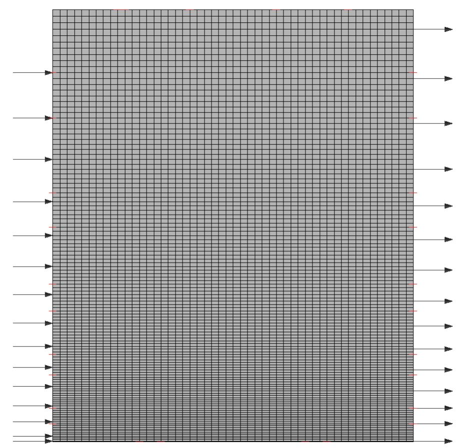

FIGURE 3. MESH OF 2D PROBLEM.

Mesh size used to calculate the work is 50 (flow) $\times$120 volumes and is shown in Fig. 3. Formula (7) in 2D case takes the form

$$\delta(U) = \frac{U}{\pi\omega\rho_m h A^2 L_x L_y} = \frac{U \cdot T}{2\pi^2 \rho_m h A^2 L_x L_y}, \tag{8}$$

where L_x is considered as the plate width in the flow direction, and L_y is characteristic length in out-of-plane direction.

Calculated amplification rate δ for 2D problem is shown in Fig. 4 together with results of [8]. One can see that curves corresponding to modes 1, 2 and 3 from analysis presented here and from [8] are in a good agreement. Absolute values of δ are different, but flutter boundaries (Mach numbers M_n^*, M_n^{**} where $\delta(M) = 0$, such that n-th mode is unstable at $M_n^* < M < M_n^{**}$) shown in Table 1 are close. A little worse correlation is observed at $M \approx 1$. This is explained by the fact that in the present study we assumed that the real part of each eigenfrequency is not changed by the flow. This condition is satisfied worse as M tends to 1. In Fig. 5 shown are real parts of calculated frequencies [8] and the plate frequencies in vacuum.

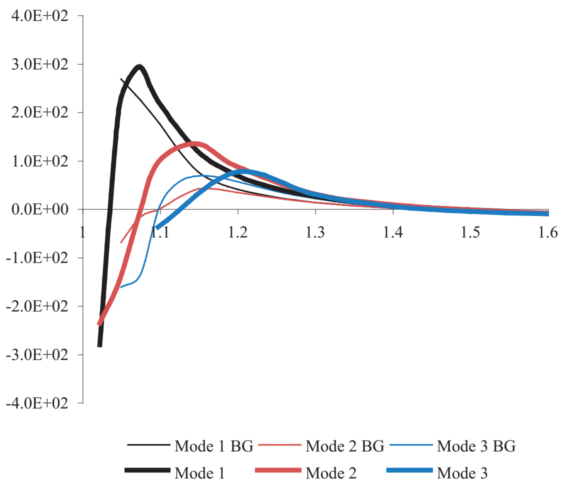

FIGURE 4. $\delta(M)$ (Hz) FOR 2D PROBLEM. "BG" DENOTES DATA FROM [8], OTHER CURVES ARE FROM THE PRESENT STUDY

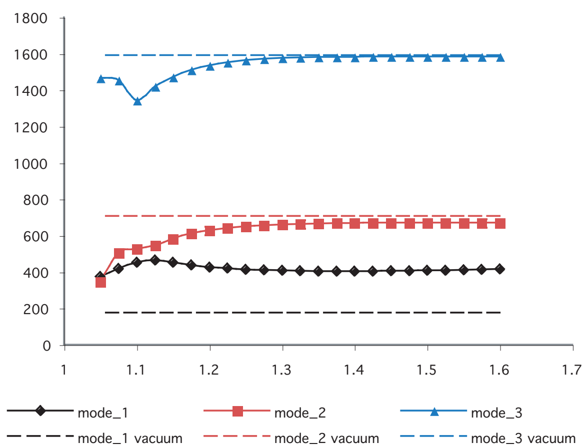

FIGURE 5. REAL PARTS OF FREQUENCIES (Hz) VS MACH NUMBER FOR 2D PROBLEM IN AIR FLOW AND IN VACUUM.

Asymptotic analysis [6] yields its unique instability criteria for each plate eigenmode. Namely, if

$$\omega_n = \sqrt{\frac{D}{\rho_m h}} \left(\frac{\pi n}{L_x}\right)^2$$

is circular frequency of the n-th mode, this mode is unstable at $M_n^* < M < M_n^{**}$, where

$$M_n^* = 1 + \sqrt{\lambda_n}, \quad M_n^{**} = \sqrt{1 + \lambda_n + \sqrt{4\lambda_n + 1}}, \qquad (9)$$

TABLE 1. FLUTTER BOUNDARIES M_n^*, M_n^{**}.

n	M_n^* pr.	M_n^{**} pr.	M_n^* [8]	M_n^{**} [8]	M_n^* [6]	M_n^{**} [6]
1	1.03	1.43	<1.05	1.42	1.05	1.42
2	1.07	1.45	1.10	1.43	1.10	1.42
3	1.12	1.44	1.10	1.44	1.15	1.44
4	—	—	1.20	1.45	1.20	1.45

$$\lambda_n = \sqrt{\frac{D}{\rho_m a^4 h}} \omega_n$$

In this formula a is speed of sound of the gas flow. Results are shown in Table 1.

Thus single mode flutter boundaries obtained in the present study are in a pretty good agreement with analytical [6] and numerical [8] results and therefore this method can be used in 3D problem study.

RESULTS

Simulation domain is shown in Fig. 2. Mesh size used to calculate the work is 52 (flow) $\times 76$ (normal to the plate) $\times 50$ volumes and is shown in Fig. 6.

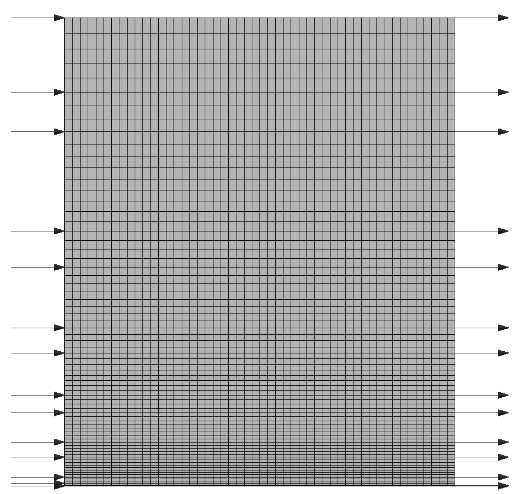

FIGURE 6. MESH OF 3D PROBLEM. CUT VIEW NORMAL TO THE PLATE IS SHOWN.

PLATE IN A HOMOGENEOUS FLOW

A set of simply supported steel plates in air flow were considered. Plate sizes were varied from 150×270 to 600×1080 m, ratio of side lengths was keeping constant. Thickness of all plates was 0.001 m. Material properties are: $E = 2 \cdot 10^{11}$ Pa, $\rho = 7800$ kg/m^3, air flow parameters are: $p = 101000$ Pa, $T = 273$ K, $a = 330$ m/s, air velocity was varied.

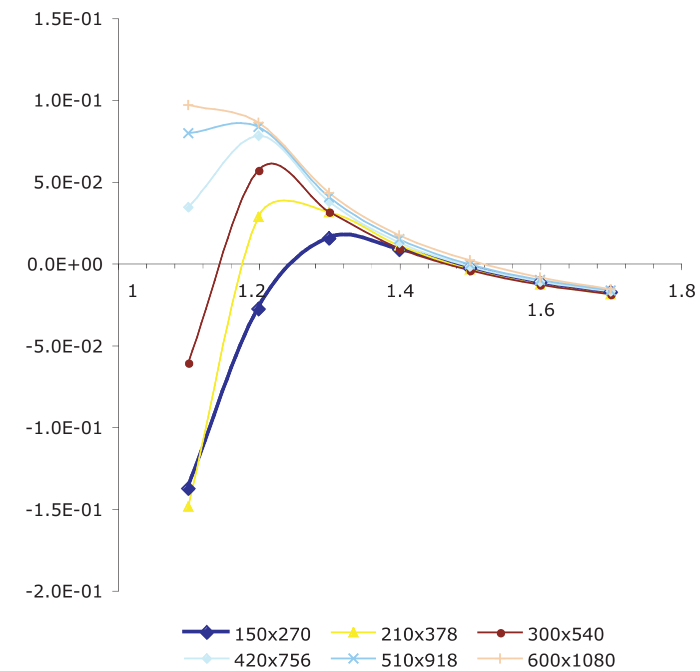

FIGURE 7. WORK DONE BY PRESSURE (kg·m^2/s^2) VS MACH NUMBER FOR THE (2,1) MODE.

In Fig. 7 shown are results of analysis for the mode (2,1) (the first number is quantity of semi-waves along spanwise direction, second number is quantity of semi-waves along chordwise direction). Comparison of the obtained single mode flutter boundaries with analytical results [7] is presented in Table 2. One can see that difference between present numerical and analytical [7] flutter boundaries is very small both for upper and lower critical Mach numbers.

INFLUENCE OF BOUNDARY LAYER

Effect of the boundary layer has been previously studied in [1]. It was noticed that appearance of the boundary layer reduces region of instability in the parameter space and can even make the plate stable. That is why in addition to studies of the plate stability in a homogeneous air flow, boundary layer influence has been also studied in this paper. Velocity profile at the inlet well as initial condition for the analysis was specified as follows:

TABLE 2. FLUTTER BOUNDARIES $M^*_{2,1}$, $M^{**}_{2,1}$ FOR (2,1) MODE.

Plate size	$M^*_{2,1}$ pres.	$M^{**}_{2,1}$ pres.	$M^*_{2,1}$ [7]	$M^{**}_{2,1}$ [7]
150×270	1.24	1.47	1.25	1.51
210×378	1.17	1.46	1.19	1.49
300×540	1.14	1.51	1.14	1.48
420×756	< 1.10	1.52	1.11	1.47
510×918	< 1.10	1.53	1.10	1.47
600×1080	< 1.10	1.54	1.09	1.47

$$v(y) = v_\infty \arctan\left(\frac{63.65y}{d}\right)\frac{2}{\pi},$$

where d is the boundary layer thickness. Inside the layer, velocity changes from 0 at $y = 0$ (plate surface) to $0.99v_\infty$ at $y = d$.

Results of the analysis are shown in Fig. 8 for the plate of 300×540 size, mode (2,1), at $M = 1.2$. Boundary layer thickness dramastically decreases the work, which in its turn is proportional to the amplification rate. Critical layer thickness is $d = 0.01$ m, the plate becomes stable at this or higher d.

CONCLUSIONS

Single mode flutter of rectangular plate is studied using Ansys CFX finite volume code. Boundary mesh nodes were moved in order to model plate eigenmode. Work done by pressure during last oscillation period is calculated. Sign of this work is the flutter criterion.

In order to verify the method, results of 2D problem were compared with papers [6] and [8]. Flutter boundaries are in a good agreement between those two papers and the present study.

For rectangular plates (3D problem), flutter boundaries are presented for several plate sizes and compared with analytical results [7]. Maximum difference between critical Mach number is less than 0.07 for both upper and lower flutter boundaries.

Boundary layer influence on the flutter boundaries is studied for $300 \times 540 \times 1$ plate. It is shown that boundary layer thickness ≈ 0.01 m is enough to suppress single mode flutter at $M = 1.2$.

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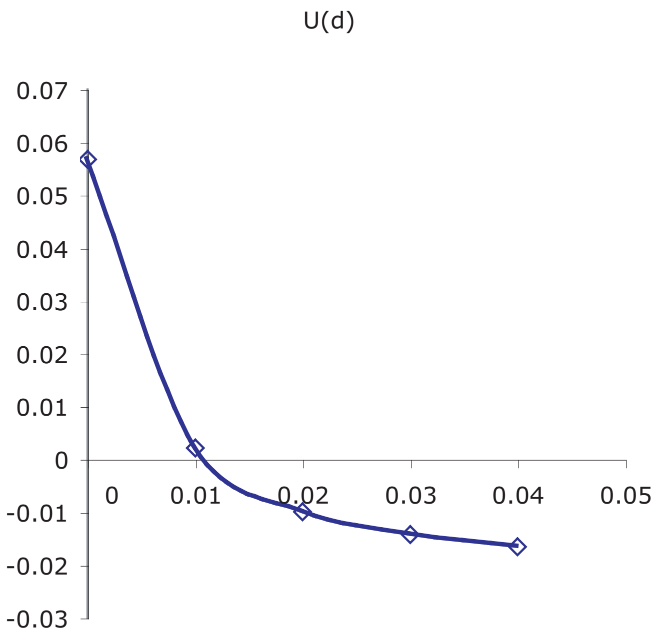

FIGURE 8. WORK DONE BY PRESSURE (kg·m^2/s^2) VS BOUNDARY LAYER THICKNESS (m) FOR THE (2,1) MODE, $M = 1.2$.

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