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On absolute instability of free jets

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Abstract. We analyze the nature of instability (absolute or convective) of a jet velocity profile obtained in preceding experimental studies. It is surprisingly found that local instability of the velocity profile near the orifice is absolute. Physical mechanism of absolute instability is revealed. Hence, absolute instability of a jet does not require counterflow, as was considered before. However, the instability observed in experiments is convective due to rapid spreading of the jet downstream from the orifice and the change of instability nature from absolute to convective. Possible methods of the prolongation of local absolute instability and its applications are discussed.

1. Introduction

The instability of fluid flows and other physical systems can be of two kinds. In the first case, when localized growing disturbances (wave packets) are carried by a flow from any given region, instability is called convective, and if they grow upstream and downstream, then absolute [1] (figure 1). It is known that in jets and wakes with "classical" velocity profiles, instability is always convective, and it can become absolute only in the presence of a sufficiently strong counterflow [2, 3]. In particular, in the flow around a cylinder the onset of the Karman vortex street for Re> 47 is explained as the result of the absolute instability of stationary cylinder wake with sufficiently strong counterflow [4].

The absolute instability can be interpreted as 1:1 internal resonance between two waves, one traveling downstream and the other upstream. In this study we show that for the case of a jet the resonance condition consists of two requirements: first, growing downstream-travelling wave governed by the inflection-point mechanism, should travel sufficiently slow, which needs sufficiently small velocity at the inflection point of the jet velocity profile. Second, the growth rate of this wave should be sufficiently large, which needs sufficiently steep velocity drop in the neighborhood of the inflection point. For classical jet profiles, both conditions are satisfied only for the jet with counterflow. However, we demonstrate an example of velocity profile without counterflow, where both conditions are satisfied, and the instability is absolute. This means that the counterflow is not necessary for absolute instability. Possible applications of this finding are discussed.

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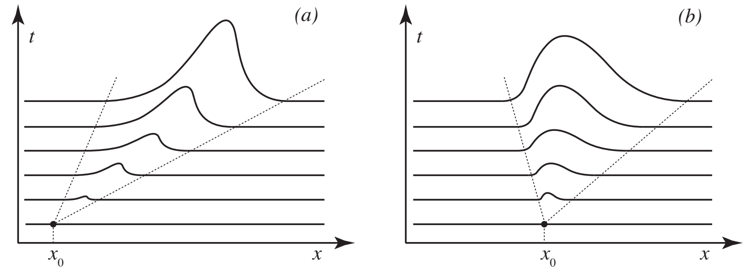

Figure 1. Convective (left) and absolute (right) instability.

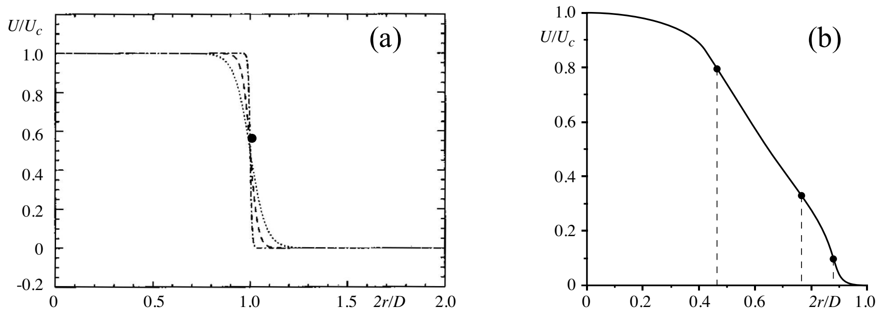

Figure 2. Classical velocity profile (1) [2,3] for R_u=1 (left) and profile generated by the device [6] (right); circles indicate the inflection point location.

2. Problem formulation and method of solution

2.1. Formulation of the problem

We consider the stability of an axisymmetric inviscid jet flow with the velocity distribution $u_r=u_\varphi=0$, $u_z=U_0(r)$, where r, φ, z are cylindrical coordinates, and the flow velocity $U_0(r)=0$ for $r>1$. As this flow always have an inflection point, such a flow is always unstable [5], i.e. there exists a real wave number α such that $\operatorname{Im}\omega(\alpha)>0$, where ω is the complex frequency. To analyse the nature of the instability, the following criterion can be used [1,2]: the instability is absolute if and only if:

- There exists a saddle point α_s of the function $\omega(\alpha)$, i.e. $d\omega/d\alpha=0$, such that $\operatorname{Im}\omega(\alpha_s)>0$.
- The point $\omega_s=\omega(\alpha_s)$ is the branch point of the reversed function $\alpha(\omega)$. In this point one of the two merging branches $\alpha(\omega)$ must correspond to a downstream, and the other to the upstream travelling wave.

If any of these two conditions is not satisfied, then the instability is convective. The nature of instability was studied for the family of jet profiles

$$u(r)=\frac{U_0}{2}\left(1+R_u\tanh\frac{R-r}{2\theta}\right) \quad (1)$$

for planar [2] and axisymmetric [3] flows (figure 2a). It was shown that the instability is absolute only for $R_u>R_{cr}>1$, i.e. when there is a sufficiently strong counterflow.

In this paper we investigate the nature of instability for the velocity profile experimentally obtained in a special device generating long laminar jets [6] (figure 2b).

2.2. Boundary-value problem for the Rayleigh equation

To analyse the instability properties of the jet with the given velocity profile (figure 2b), we need to calculate the $\alpha(\omega)$ function, which is performed numerically by solving the boundary-value problem for the axisymmetric Rayleigh equation. For each complex frequency ω, the eigenvalue $\alpha(\omega)$ is found iteratively by the secant method. At each iteration the boundary-value problem is reduced to two initial-value problems, each of them is solved by Runge-Kutta method. Detailed description of the numerical procedure is omitted here for the sake of brevity and can be found in [6].

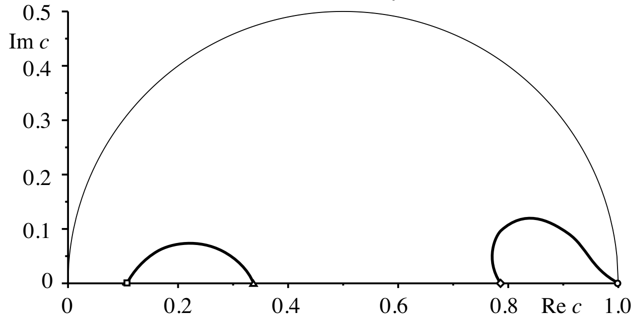

Figure 3. Two branches of temporarily growing perturbations.

3. Results

3.1. Temporal and spatial instability

The velocity profile (figure 2b) has three inflections points, and each of them produces instability [6]. Figure 3 shows results of the temporal instability analysis; namely, images of the real α on the complex phase plane $c=\omega/\alpha$ are shown (we consider only growing perturbations, Im $c>0$). It is seen that there are two branches of growing perturbations. The first branch connects two neutral perturbations produced by the outer and the intermediate inflections points; the second branch connects neutral perturbation produced by the inner inflection point and $c=1$.

Figure 4 shows the results of spatial instability analysis: spatial growth rates −Im α versus real ω. It is seen that while for the second branch the growth rate curve is continous and corresponds to the second temporal branch, the first branch is discontinuous: start from one neutral perturbation does not lead to the second neutral perturbation.

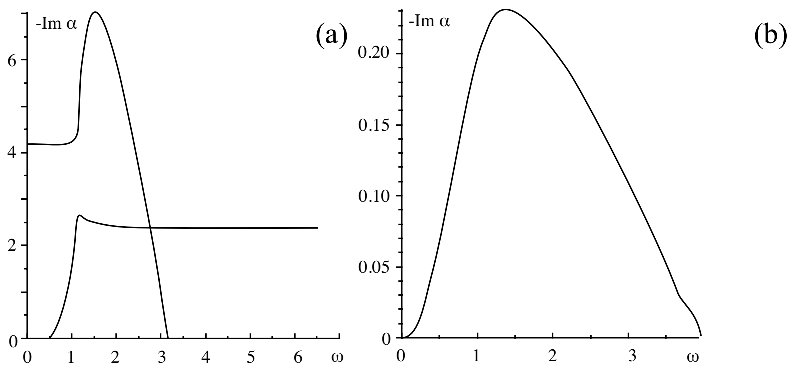

Figure 4. Spatial growth rates versus real frequency for the first (a) and second (b) branches of growing perturbations.

3.2. The onset of absolute instability

To better investigate the reason of such a behavior, we consider images α(ω) of different lines Im ω=const. It can be seen in figure 5 that while for Im ω=0.05 the first branch curve continuously connects two neutral perturbations, for Im ω=0.02 this branch interacts with another branch of damped, upstream-travelling wave perturbations, which produces saddle point of the ω(α) function. Detailed calculations yield $\omega_s \approx 1.112+0.039i$, $\alpha_s \approx 2.3-3.3i$, which means that the flow is absolutely unstable. However, this result contradicts previously known result for the velocity profiles [2, 3], as the instability of the jet without counterflow is always convective. Also, if the instability is absolute, it should completely destroy the steady laminar flow; however, a long laminar jet with this velocity profile is observed in experiments [6]. Below these two contradictions with expected result are discussed in more details.

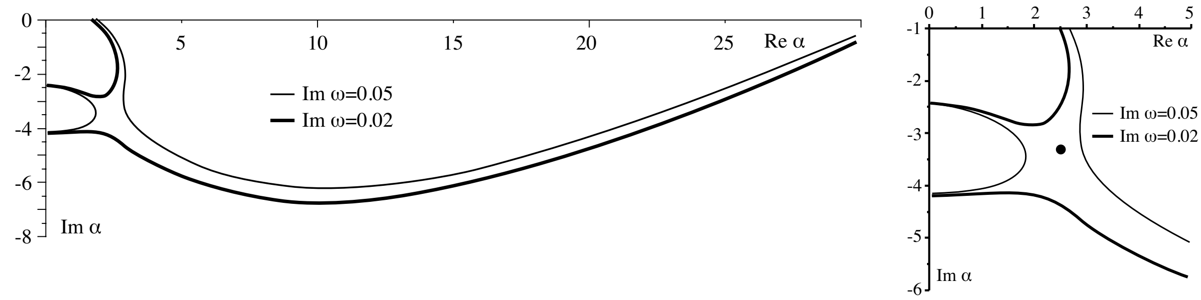

Figure 5. Segments of lines Im ω=const under the action of α(ω) function (left), enlargement of the saddle point area (right).

3.3. Physical mechanism of the absolute instability

To investigate the mechanism of absolute instability, let us consider the eigenmodes corresponding to damped upstream-travelling mode and growing downstream-travelling mode shown in figure 6. It is seen that growing eigenmode is concentrated around the inflection point. Because of this, its phase speed Re c is close to the phase speed of neutral mode, i.e. to the flow velocity in the inflection point [5]. Also, it can be shown that the growth rate is proportional to the slope of the velocity profile near the inflection point. Hence, both phase speed and growth rate are governed only by local velocity distribution near the inflection point and can be changed by local deformation of the velocity profile.

On the contrary, the damped eigenmode is essentially non-zero in the jet (figure 6). Hence, local changes of the velocity profile cannot affect this mode.

Essential property of the damped upstream-travelling mode is that in a certain range of frequencies it is convected by the flow and becomes downstream-travelling (figure 5). If the phase speed of the "true" downstream-travelling mode is sufficiently low, and the growth rate is sufficiently large, this yields their coalescence that can be interpreted as internal 1:1 resonance, or the absolute instability.

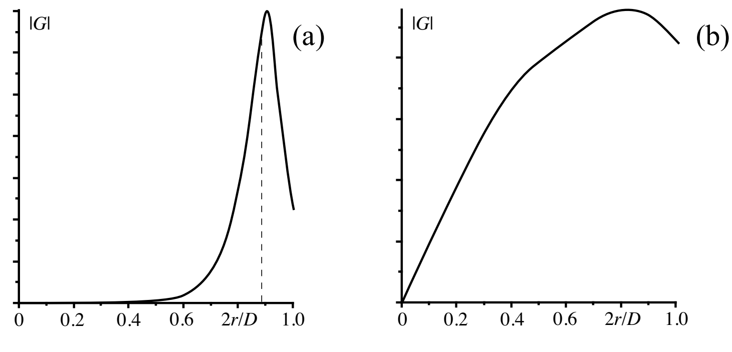

Figure 6. Eigenmode amplitude for the downstream (a) and upstream (b) travelling mode.

Taking these considerations into account, observation of figure 2a shows that phase speed of the growing mode is c≈0.5, whereas the upstream-travelling wave in the frequency range where it becomes downstream-travelling cannot reach such a large travel speed. On the contrary, the profile shown in figure 2b has much smaller velocity in the outer inflection point equal to the phase speed of the growing mode (~0.1), and quite a large slope near the inflection point. They both are reached by the upstream-travelling wave, which yields the absolute instability of this profile.

3.4. Downstream evolution of the instability nature

As discussed in section 3.3, absolute instability appears due to small velocity and large slope of the velocity profile at the inflection point. However, due to the action of viscosity, the jet slightly spreads downstream near its outer boundary (figure 7a), and the absolute instability is changed by convective at the distance of just 0.5D (where D is the jet diameter). Further downstream, at the distance 1.5D, two outer inflection points merge and disappear (figure 7b, c), as well as the instability produced by these two points.

This mechanism of jet spreading explains the absence of the absolute instability in experiments, because global eigenmode of the evolving jet profile convects dowstream due to too small segment of the local absolute instability.

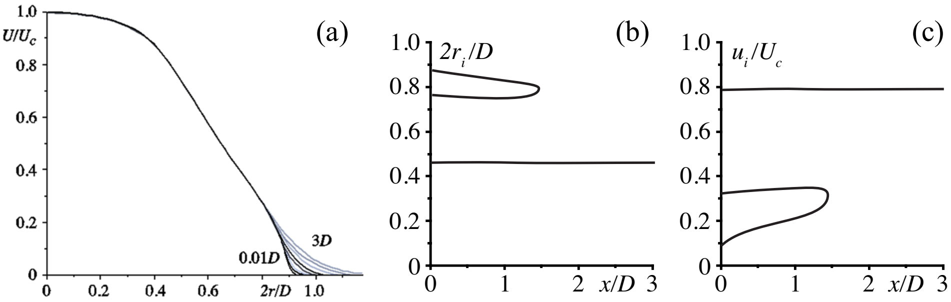

Figure 7. Spreading of the velocity profile downstream from the orifice (a). Location of inflection points (b) and velocity at inflection points (c).

4. Concluding remarks

In this paper, we have shown that the jet velocity profile not having counterflow and emerged from the experiment [6] is absolutely unstable. Physical mechanism of absolute instability is revealed, which explains the difference of this jet profile from classical profiles (1), where the jet without counterflow is always convectively unstable.

However, the local absolute instability is not observed in experiment, because of viscous spreading of the steady profile downstream from the orifice, and rapid change of local instability nature from absolute to convective.

To eliminate the jet spreading and, hence, to prolong the absolute instability, special devices can be used. For example, splitter plates with the jet-diameter orifices can be installer to prevent involving surrounding air into the axial motion. Alternatively, suction of the flow around the jet can be organized.

The prolongation of absolute instability can result in the global absolute instability of the spatially evolving jet, which yields its turbulization immediately at the orifice (not at some point downstream from the orifice). Such a "total" turbulization can be used in various technologies to enhance mixing, for example, in fuel injectors.

Acknowledgments
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